INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN PHYSICS 2013-2014

Date of Examination : 24th November 2013

Time 09.30 to 11.30 hrs

Total time : 120 minutes (A-1, A-2 & B)

[Q.P. CODE NO. : 1-3-5]

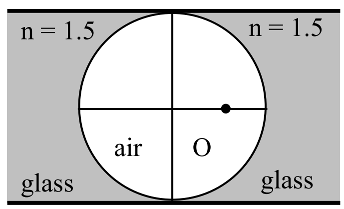
PART-A **(Total Marks : 180)**

SUB-PART A-1 : ONLY ONE OUT OF FOUR OPTIONS IS CORRECT

N.B. Physical constants are given at the end

SUB-PART A-1

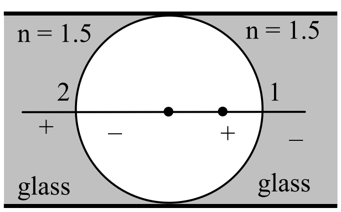
1. Two concave refracting surfaces of equal radii of curvature face each other in air as shown in figure. A point object O is placed midway between the centre and one of the poles. Then the separation between the images of O formed by each refracting surface is



(a) 11.4 R (b) 1.14R (c) 0.114R (d) 0.0114 R

Ans. [c]

Sol.



I_1 due to refraction at surface (1)

$$\frac{1.5}{V} - \frac{1}{R/2} = \frac{1.5-1}{R}$$

$$\frac{1.5}{V} = \frac{2}{R} + \frac{0.5}{R}$$

$$\frac{1.5}{V} = \frac{2.5}{R}$$

$$V = \frac{15}{25}R \Rightarrow \frac{3}{5}R = 0.6R$$

I_2 due to refraction at surface (2)

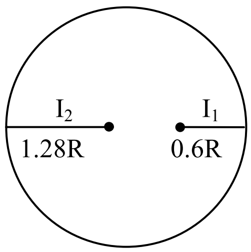
$$\frac{1.5}{V} - \frac{1}{-3R/2} = \frac{1.5-1}{-R}$$

$$\frac{1.5}{V} = -\frac{2}{3R} - \frac{0.5}{R}$$

$$\frac{1.5}{V} = \frac{-3.5}{3R}$$

$$V = \frac{-4.5}{3.5}R \Rightarrow -\frac{9}{7}R$$

$\Rightarrow -1.285$ R



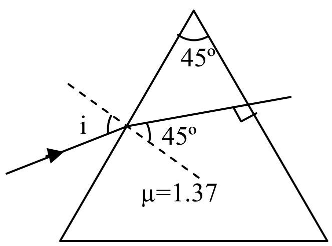
distance between I_1 & I_2 = 0.114 R

2. A ray of white light falls on an isosceles prism at such an angle that the red light leaves the prism perpendicular to the other face of the prism. Find angle of deviation if the refractive index of the prism for red light is 1.37 and refracting angle of prism is 45º

(a) 20º37′ (b) 28º37′ (c) 35º37′ (d) 30º37′

Ans. **[d]**

Sol.



$\sin i \times 1 = 1.37 \sin 45^\circ$

$\sin i = 0.968$

$i = \sin^{-1}(0.968)$

$i = 75.6^\circ$

$\delta = i + e - A$

$= i - A$

$\Rightarrow 75.6 - 45$

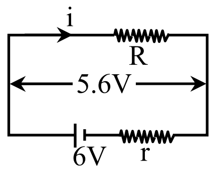
$\Rightarrow 30.37'$

3. The voltage between the terminals of a battery is 6.00 V. When a wire is connected across its terminals it falls to 5.6 V. If one more identical wire is connected between the terminals then it will fall to

(a) 4.80 V (b) 5.15 V (c) 5.25 V (d) 5.80 V

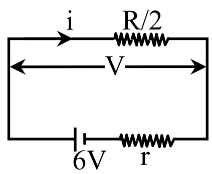
Ans. **[c]**

Sol.



$iR = 5.6$

$$\frac{6R}{R+r} = 5.6 \qquad \ldots(i)$$



$$i'\frac{R}{2} = V$$

$$\frac{6R}{R+2r} = V \qquad \ldots(ii)$$

from (i) & (ii)

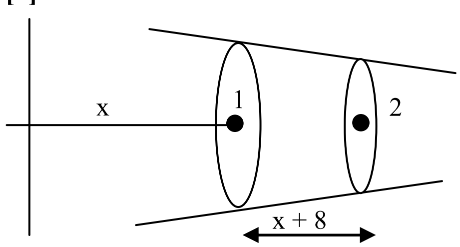
$V = 5.25$

4. The radius of cross section of a long pipe varies gradually as $r = r_0e^{-\alpha x}$, where x is the distance from the pipe inlet and $\alpha = 0.4\ m^{-1}$ is a constant. Then the ratio of Reynolds number for the two cross sections separated by a distance 8.0 m is

(a) 24.5 (b) 28.5 (c) 2.45 (d) 2.85

Ans. **[a]**

Sol.



$A_1v_1 = \Delta_2v_2$

$$R_e = \frac{Dv\rho}{\mu}$$

$$\frac{R_{e_1}}{R_{e_2}} = \frac{R_1}{R_2} \times \frac{v_1}{v_2}$$

$$= \frac{R_1}{R_2} \times \frac{A_2}{A_1}$$

$= \frac{R_1}{R_2} \times \frac{R_2^2}{R_1^2}$

$= \frac{R_2}{R_1}$ [Ratio of radius]

$= \frac{r_0 e^{-0.4(x+8)}}{r_0 e^{-0.4x}}$

$= e^{-0.4x - 0.4 \times 8 - (0.4x)}$

$\frac{R_{e_1}}{R_{e_2}} = e^{-3.2}$

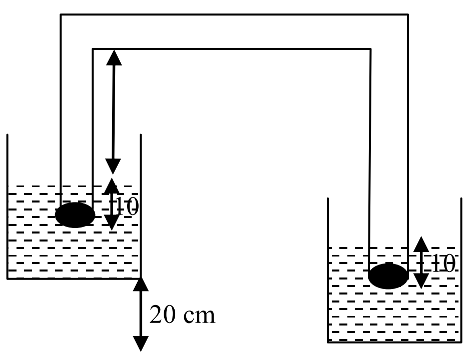
$\frac{R_{e_2}}{R_{e_1}} = 24.5$

5. Water is siphoned out from a tank at a higher level into another of identical size 2.0 m below. The length of the siphon tube is 4.0 m and each of its ends is below the water surface by 10 cm. In the upper tank the water level is at 1.00 m and in the lower one it is at 50 cm from the bases of the respective tanks. Water through siphon rises in 40 cm of the length of the tube to a level which is 20 cm higher than the water level in the upper tank before it begins to flow down. Assume the values of atmospheric pressure and acceleration due to gravity to be 103.4 kPa and 10 m/s^2. The pressure at the cross section of the tube at the highest point is

(a) 101.9 kPa (b) 105.4kPa (c) 107.4kPa (d) 109.1kPa

Ans. **[b]**

Sol.

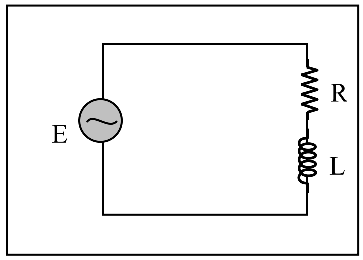


$P_0 + \rho gh = P'$

$103.4 \times 10^3 + 10 \times 1000 \times (0.2) = P'$

$P' = 105.4 \times 10^3$

6. The impedance of the RL circuit given in the adjacent figure is expressed by the relation $Z^2 = A^2 + B^2$. Then the dimensions of AB are



(a) $[M^1L^2I^{-2}T^{-3}]$ (b) $[M^2L^4I^{-4}T^{-6}]$ (c) $[M^1L^{-1}I^{-2}T^{-3}]$ (d) $[M^{-1}L^{-2}I^{+2}T^4]$

Ans. **[b]**

Sol. $Z = \sqrt{R^2 + X_L^2}$

$Z^2 = R^2 + X_L^2$

unit of A & B is Ω unit of AB is Ω^2

$P = I^2R$

unit of Ω $\frac{P}{I^2} \Rightarrow \frac{MLT^{-2}LT^{-1}}{I^2}$

$\Rightarrow M^1L^2T^{-3}I^{-2}$

unit of AB is $\Omega^2 \Rightarrow M^2L^4T^{-6}I^{-4}$

7. A micrometer screw gauge with pitch of 0.5 mm and 50 divisions on circular scale is used to measure the diameter of a thin wire. Initially when the gap is closed the line of fourth division conicides with the reference line. Three readings show 46^{th}, 48^{th} and 44^{th} division coinciding with the reference line which is beyond 0.5 mm of the main scale. The (best) measured value is

(a) 0.46 mm (b) 0.94 mm (c) 0.92 mm (d) 1.00 mm

Ans. **[c]**

Sol. L.C of circular scale = $\frac{0.5}{500}$ = 0.01 mm

error = + 0.04

correction = – 0.04

Reading

$R_1 = 0.5 + 46 \times 0.01 - 0.04$

$\Rightarrow 0.96 - 0.04 = 0.92$

$R_2 = 0.5 + 48 \times 0.01 - 0.04$

$\Rightarrow 0.98 - 0.04 = 0.94$

$R_3 = 0.5 + 44 \times 0.01$

$\Rightarrow 0.94 - 0.04 = 0.90$

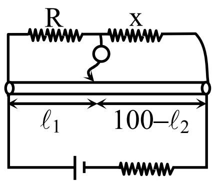
Average reading = 0.92 mm

8. In a meter bridge experiment the resistance to be measured is connected in the right gap and a known resistance in the left gap has value of 50 ± 0.2 Ω when the null points is judged to be at 60 ± 0.2 cm. The students notes that the ends of the bridge wire are not at 0.0 cm and 100.0 cm of the scale and makes a guess that they may be somewhere within 0.2 cm beyond the scale ends. The value of the unknown resistance should be expressed as

(a) 33.33 ± 1 Ω (b) 75 ± 1 Ω (c) 75.0 ± 0.9 Ω (d) 33.4 ± 0.5 Ω

Ans. **[d]**

Sol.



$R_0 = 50 \pm 0.2\Omega$

$R = 50\Omega$ & $\Delta R = 0.\ 2\Omega$

$\ell_0 = 60 \pm 0.2$ cm

$\ell = 60$ cm, $\Delta\ell = 0.2$ cm

$\ell_1 \pm 0.2$ cm & $(100 - \ell_1) \pm 0.4$ cm

$$x = \frac{R(100 - \ell_0)}{\ell_0}$$

$$x = \frac{50 \times 40}{60} = \frac{200}{6} = 33.33$$

$\%x = \%R + 2\%\ \ell_0$

$$100 \times \frac{\Delta x}{x} = \frac{0.2}{50} \times 100 + 2 \times \frac{0.2}{60} \times 100$$

$$= 0.4 + 0.66 = \frac{1.06}{100} \times x = \Delta x$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{6+10}{15}$$

$$= \frac{16}{15} \times \frac{100}{3 \times 100} = \frac{16}{45}$$

9. Suppose a radioactive nucleus A disintegrates at origin into a nucleus B with the emission of a positron (e^+) and a neutrino (ν) such that the positron and the neutrino move at right angles to each other and carry momentum 2×10^{-22} kg ms^{-1} (+Y-axis) and 5×10^{-23} kg ms^{-1} (+X-axis) respectively. Then the nucleus

(a) A recoils with the momentum 2.86×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. + X-axis

(b) A recoils with the momentum 2.06×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. – X-axis

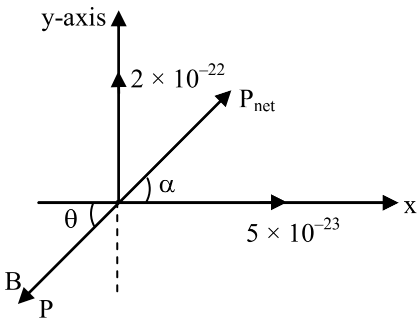
(c) B recoils with the momentum 2.86×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. + X-axis

(d) B recoils with the momentum 2.06×10^{-22} kg ms^{-1} at angle 14.03º w.r.t. – X-axis

Ans. **[d]**

Sol. A radioactive have momentum = 0

when it disintegrate at origin then momentum after disintegration remain zero.



P is the vector sum of momentum of e^+ and v.

$P = \sqrt{(2\times10^{-22})^2 + (5\times10^{-23})^2}$

Momentum of B = P = 2.06×10^{-22}

$P \cos\theta = 5 \times 10^{-23}$

$P \sin\theta = 2 \times 10^{-22}$

$\tan\theta = \frac{2}{5} \times 10 = 4$

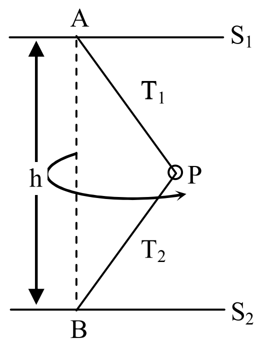
$\theta = \tan^{-1}4 = 75.96$ with negative x-axis.

No answer match with any given option.

However magnitude of P match with option 'd'

So option 'd' is correct.

10. Two identical strings with fixed ends separated by height h have their other ends tied to a body P of mass m as shown in figure. When the body rotates with uniform angular speed $2\sqrt{2g/h}$ in a horizontal plane about the vertical axis the ratio of tensions (T_1/T_2) in the string is



(a) 3/5 (b) 5/3 (c) 2/5 (d) 5/2

Ans. **[b]**

Sol. $T_1 \cos\theta = mg + T_2 \cos\theta$

$(T_1 - T_2)\cos\theta = mg$

$(T_1 + T_2)\sin\theta = m\omega^2 r$

$\frac{T_1 + T_2}{T_1 - T_2} \times \tan\theta \Rightarrow \frac{\omega^2 r}{g}$

$$\frac{T_1+T_2}{T_1-T_2}\times\frac{2r}{h}=\frac{4\times 2g}{h}\times\frac{r}{g}$$

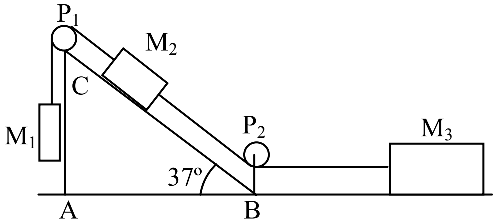
$$\frac{T_1+T_2}{T_1-T_2}=4$$

$T_1 + T_2 = 4T_1 - 4T_2$

$5T_2 = 3T_1$

$$\left(\frac{T_1}{T_2}=\frac{5}{3}\right)$$

11. Two identical bodies M_2 and M_3 each of 4.0 kg are tied to a massless inextensible string which is made to pass around pulleys P_1 and P_2 as shown in figure. Angle ABC = 37º. The coefficient of kinetic friction between the bodies and the surface on which they slide is 0.25. If the body M_1 moves down with uniform speed, neglecting the masses and friction of pulleys, M_1 =



(a) 36.8 kg (b) 9.8 kg (c) 4.2 kg (d) 2.1 kg

Ans. **[c]**

Sol. $M_1g - T_1 = 0$ ...(1)

$T_1 - M_2g \sin 37º - f - T_2 = 0$

$T_2 - \mu M_3g = 0$

$M_1g - M_2g \sin 37º - \mu\, M_2\, g \cos 37º - \mu M_3g = 0$

$$M_1 - 4\times\frac{3}{5}-\frac{1}{4}\times 4\times\frac{4}{5}-\frac{1}{4}\times 4 = 0$$

$$M_1 - \frac{12}{5}-\frac{4}{5}-1=0$$

$$M_1 \Rightarrow \frac{16}{5}+1 \Rightarrow \frac{21}{5} \Rightarrow 4.2$$

12. A particle of mass 0.2 kg moves along a path given by the relation : $\vec{r} = 2\cos\omega t\,\hat{i} + 3\sin\omega t\,\hat{j}$. Then the torque on the particle about the origin is

(a) $\sqrt{13}\ \hat{k}$ Nm (b) $\sqrt{2/3}\ \hat{k}$ Nm (c) $\sqrt{3/2}\ \hat{k}$ Nm (d) $0\ \hat{k}$

Ans. **[d]**

Sol. $\vec{r} = 2\cos\omega t\,\hat{i} + 3\sin\omega t\,\hat{j}$

$$\vec{v}=\frac{d\vec{r}}{dt} = -(2\omega\sin\omega t)\,\hat{i} + (3\omega\cos\omega t)\,\hat{j}$$

$\vec{a} = \dfrac{d^2r}{dt^2} = -(2\omega^2 \cos\omega t)\,\hat{i} + (-3\omega^2 \sin\omega t)\,\hat{j}$

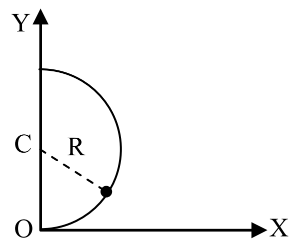
$= -\omega^2(2 \cos \omega t\,\hat{i} + 3 \sin \omega t\,\hat{j})$

$\vec{a} = -\omega^2\vec{r}$

$\Rightarrow \vec{F} = -\omega^2 m\vec{r}$

$\vec{\tau} = \vec{r} \times \vec{F} = 0$

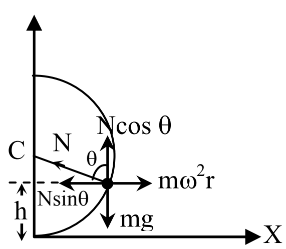
13. A bed of mass 5.0 g can move without friction on a piece of wire bent in the form of a semicircular ring of radius 0.10 m, as shown in the adjacent figure. This ring can freely rotate about the vertical axis OY. At what height will the bead stay above the around level OX, if this semicircular arc revolves with angular velocity 10.63 rad/s ?



(a) 0.013 m (b) 0.087 m (c) 0.027 m (d) 0.073 m

Ans. **[a]**

Sol.



$h = R - R \cos \theta$ ...(1)

$N \cos \theta = mg$

$N \sin \theta = m\,\omega^2\, r$

$N \sin \theta = m\,\omega^2(R \sin \theta)$

$N = m\,\omega^2 R$

$m\omega^2 R \cos \theta = mg$

$h = R - R \times \dfrac{g}{\omega^2 R}$

$\Rightarrow R - g/\omega^2$

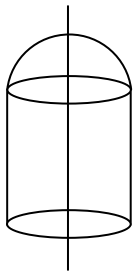
$\Rightarrow 0.1 - \dfrac{10}{(10.63)^2} = 0.013$

14. One of the flat surfaces of a cylinder (radius r and length ℓ) and the flat surface of a hemisphere are cemented together. The cylinder and the hemisphere are made of the same material. The combined mass of the system is M. The moment of inertia of this system about an axis coinciding with the axis of cylinder is

(a) $(1/10)\ M\{5r^2 + 4\ell^2\}$
(b) $(1/10)\ Mr^2\{(15\ell + 8r)/(3\ell + 2r)\}$
(c) $1/10\ Mr^2\ \{(3\ell + 4r)/(3\ell + 2r)\}$
(d) $(1/10)\ M\{5r^3\ell + 4\ell^2r^2)/(5r + 4\ell)\}$

Ans. **[b]**

Sol.



$m = m_1 + m_2$

$M = m_1 + m_2$

$$\frac{m_1}{m_2} \Rightarrow \frac{\rho \times 2/3\pi R^3 \rho}{\rho \times \pi R^2 . \ell}$$

$$1 + \frac{m_1}{m_2} \Rightarrow \frac{2}{3} . \frac{R}{\ell} + 1$$

$$\frac{m}{m_2} \Rightarrow \frac{2R + 3\ell}{3\ell}$$

$$m_2 \Rightarrow \frac{3\ell m}{(2R + 3\ell)}$$

$$m_1 = m - \frac{3m\ell}{2R + 3\ell}$$

$$m_1 = \frac{2Rm}{2R + 3\ell}$$

$I = I_1 + I_2$

$$= \frac{2}{5} m_1 R^2 + \frac{m_2}{2} R^2$$

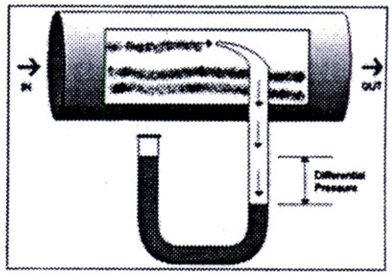
$$I = R^2\left[\frac{2m_1}{5} + \frac{m_2}{2}\right]$$

$$= R^2 m\left[\frac{2}{5} \times \left(\frac{2R}{2R + 3\ell}\right) + \frac{1}{2} \times \frac{3\ell}{(2R + 3\ell)}\right]$$

$$= \frac{mR^2}{(2R + 3\ell)}\left[\frac{4}{5}R + \frac{3}{2}\ell\right]$$

$$= \frac{mR^2}{2R + 3\ell}\left[\frac{8R + 15\ell}{10}\right]$$

15. The pitot tube shown in the figure is used to measure fluid velocity in a pipe of cross sectional area S. It was invented by a French engineer Henri Pitot in the early 18^{th} century . The volume of the gas flowing across the section of the pipe per unit time is (The difference in the liquid columns is Δh, ρ_0 and ρ are the densities of liquid and the gas respectively)



(a) $Q = 2S\sqrt{\frac{\Delta h\rho_0 g}{\rho}}$ (b) $Q = S\sqrt{\frac{2\Delta h\rho_0 g}{\rho}}$ (c) $Q = S\sqrt{\frac{\Delta hg\rho_0}{\rho}}$ (d) $Q = S\sqrt{\frac{2\Delta h\rho g}{\rho_0}}$

Ans. **[b]**

Sol. $\frac{PV^2}{2} = \Delta P$

$V = \sqrt{\frac{2\Delta P}{P}} = \sqrt{\frac{2 \times \Delta hg\rho_0}{\rho}}$

Volume flow rate = A × V

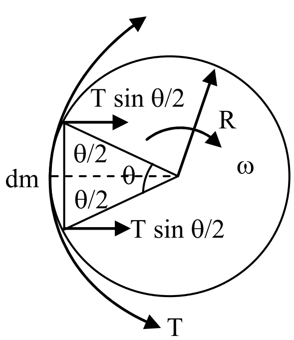
$= S \times \sqrt{\frac{2\Delta hg\rho_0}{\rho}}$

16. A thin ring has a radius R, density ρ and Young's modulus Y. The ring is rotated in its own plane about an axis through its centre with angular velocity ω. Then the small increase in its radius is

(a) $dR = \frac{\rho\omega^2 R^3}{Y}$ (b) $dR = \frac{3\rho\omega^2 R^3}{Y}$ (c) $dR = \frac{6\rho\omega^2 R^3}{Y}$ (d) $dR = \frac{\rho\omega^2 R^3}{2Y}$

Ans. **[a]**

Sol.



$2T \sin \theta/2 = dm\omega^2 R$

$T\theta = A\rho \times R\theta\, \omega^2 R$

$T = \rho R^2\omega^2 A$

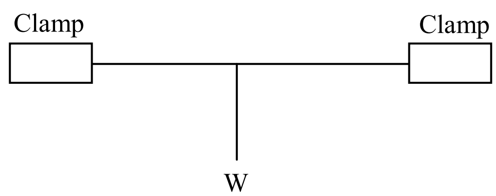
$$\text{stress} = \frac{T}{A} = \rho R^2\omega^2$$

$$\text{strain} = \frac{\rho L^2\omega^2}{Y}$$

$$\frac{2\pi\Delta R}{2\pi R} = \frac{\rho R^2\omega^2}{Y}$$

$$\Delta R = \frac{\rho R^3\omega^2}{Y}$$

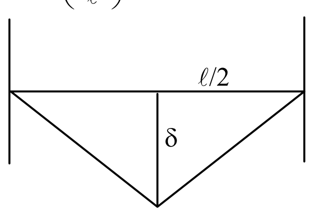
17. A uniform metal wire is clamped by chuck nuts at the two ends as shown in the adjacent figure. The wire has cross section area A, length ℓ and density ρ. A weight W is suspended from the midpoint of the wire. Then the vertical displacement δ, through which the midpoint moves down is given by



(a) $\delta = \frac{\ell}{2}\left(\frac{W}{AY}\right)^{1/2}$ (b) $\delta = \frac{\ell}{2}\left(\frac{W}{AY}\right)^{1/3}$ (c) $\delta = \frac{\ell}{3}\left(\frac{W}{AY}\right)^{1/2}$ (d) $\delta = \frac{\ell}{4}\left(\frac{W}{AY}\right)^{1/3}$

Ans. **[b]**

Sol. $Y = \dfrac{F/A}{\left(\dfrac{\Delta\ell}{\ell}\right)}$



$$Y = \frac{T.\ell}{A\ \Delta\ell}$$

$$\Delta\ell = 2\sqrt{\delta^2 + \ell^2/4} - \ell$$

$$\Rightarrow [4\delta^2 + \ell^2]^{1/2} - \ell$$

$$\Rightarrow \ell\left[1 + \frac{4\delta^2}{\ell^2}\right]^{1/2} - \ell$$

$$\Rightarrow \ell\left[1 + \frac{4\delta^2}{2\ell^2}\right] - \ell$$

$$\Delta\ell = \frac{2\delta^2}{\ell}$$

$$Y = \frac{W}{4\delta} \times \frac{1}{A} \times \frac{\ell^3}{2\delta^2}$$

$$\delta^3 = \frac{W\ell^3}{8AY}$$

$$\delta = \frac{\ell}{2}\left(\frac{W}{AY}\right)^{1/3}$$

18. Two bodies of masses M_1 and M_2 are kept separated by a distance d. The potential at the point where the gravitational field produced by them is zero, is

(a) $-\frac{G}{d}(M_1 + M_2 + 2\sqrt{M_1M_2})$

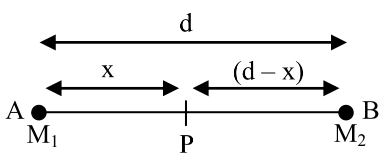
(b) $-\frac{G}{d}(M_1M_2 + 2\sqrt{M_1 + M_2})$

(c) $-\frac{G}{d}(M_1 - M_2 + 2\sqrt{M_1M_2})$

(d) $-\frac{G}{d}(M_1M_2 - 2\sqrt{M_1 + M_2})$

Ans. **[a]**

Sol.



at P, E = 0

i.e. $\frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2}$

$M_1 (d - x)^2 = M_2x^2$

$$\left(\frac{d-x}{x}\right) = \sqrt{\frac{M_2}{M_1}}$$

$$\frac{d}{x} - 1 = \sqrt{\frac{M_2}{M_1}}$$

$$\frac{d}{x} = 1 + \sqrt{\frac{M_2}{M_1}} = \frac{\sqrt{M_1} + \sqrt{M_2}}{\sqrt{M_1}}$$

$$x = \frac{\sqrt{M_1}\ d}{(\sqrt{M_1} + \sqrt{M_2})} \text{ and } (d - x) = d - \frac{\sqrt{M_1}\ d}{(\sqrt{M_1} + \sqrt{M_2})}$$

$$(d - x) = \frac{\sqrt{M_2}\ d}{\sqrt{M_1} + \sqrt{M_2}}$$

$$V = -\frac{-GM_1}{x} - \frac{-GM_2}{(d-x)}$$

$$= -G\left[\frac{M_1(\sqrt{M_1}+\sqrt{M_2})}{\sqrt{M_1}\,d} + \frac{M_2(\sqrt{M_1}+\sqrt{M_2})}{\sqrt{M_2}\,d}\right]$$

$$= -\frac{G(\sqrt{M_1}+\sqrt{M_2})}{d}\left(\sqrt{M_1}+\sqrt{M_2}\right)$$

$$= -\frac{G}{d}(M_1 + M_2 + 2\sqrt{M_1M_2})$$

19. A ball dropped on ground from a height of 1.00 m rises to a height of 75 cm on rebounce. When thrown down from the same height with a velocity of 2.0 m/s, it would rise to (assume g = 10 m/s^2)

(a) 80 cm (b) 90 cm (c) 85 cm (d) 95 cm

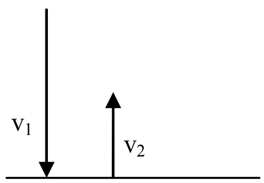
Ans. **[b]**

Sol. $v_2 = ev_1$

$\sqrt{2gh_2} = e\sqrt{2h_1g}$

$h_2 = e^2h$

$e = \sqrt{\frac{75}{100}} = \sqrt{3/4}$



$v_2' = ev_1'$

$\sqrt{2gh'} = \sqrt{\frac{3}{4}}\sqrt{4+2g\times 1}$

$\sqrt{2gh'} = \sqrt{\frac{3}{4}}\sqrt{24} \Rightarrow \sqrt{3}\times\sqrt{6}$

$\sqrt{2gh'} = \sqrt{18} \Rightarrow h' = 90$ cm

20. A metallic body of material with density of 8000 kg/m^3 has a cavity inside. A spring balance shows its mass to be 10.0 kg in air and 7.5 kg when immersed in water. The ratio of the volume of the cavity to the volume of the material of the body must be

(a) 2/5 (b) 1/2 (c) 1 (d) 3/4

Ans. **[c]**

Sol. Volume of material = V

Volume of cavity = x

A spring balance reading = 10.0 kg

$10 = V\rho$

where ρ = density of metallic body material

= 8000 kg/m^3

$10 = V \times 8000$ ....(1)

$V = \frac{10}{8000}$

A spring balance reading in mater = 7.5 kg = 75 N

$75 = 10 \times 10 - f_{Buoyant}$

$f_{Buoyant} = 100 - 75 = 25$ N

$f_{Buoyant} = (v + x)\rho_{water} \times g$

$(v + x) \times 1000 \times g = 25$

$v + x = \frac{25}{10000}$

$\frac{v+x}{v} = \frac{25}{10000} \times \frac{8000}{10}$

$\frac{v+x}{v} = \frac{2}{1}$ $\quad \therefore v + x = 2v$

$\frac{x}{v} = 1$

21. In a steel factory it is found that to maintain M kg of iron in the molten state at its melting point an input power P watt is required. When the power source is turned off, the sample completely solidifies in time t second. The latent heat of fusion of iron is

(a) $\frac{2Pt}{M}$ (b) $\frac{Pt}{2M}$ (c) $\frac{Pt}{M}$ (d) $\frac{PM}{t}$

Ans. **[c]**

Sol. Initially it was maintained at molten state by supplying power ρ. It mean from in loosing heat at the rate of P watt.

$\therefore$ When power is switch off. It looses heat at the rate of P watt.

In time t heat loss = $P \times t$

In time t it get solidify

$\therefore P \times t = ML$

$L = \frac{Pt}{M}$

22. A LASER source of heat of power 1.2 W is placed very close to one end of a rod of cross section area 3 cm^2 and thermal conductivity 400 W/mK. The length of the rod (L) required to maintain a temperature difference of 10^0C across its ends is (assume that all power emitted by the source falls on the rod).

(a) 1.5 m (b) 2.2 m (c) 1.8 m (d) 1m

Ans. **[d]**

Sol. $\frac{dH}{dt} = \frac{\Delta T}{L/KA}$

$\frac{dH}{dt} = 1.2$ watt

$\Delta T = 10$

$K = 400$ W/mk

$$1.2 = \frac{10 \times 400 \times 3 \times 10^{-4}}{L}$$

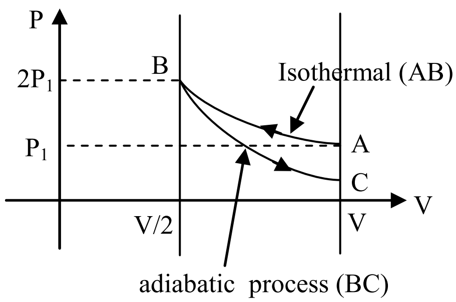
L = 1 m

23. A certain quantity of oxygen ($\gamma = 7/5$) is compressed isothermally until its pressure is doubled (P_2). The gas is then allowed to expand adiabatically until its original volume is restored. Then the final pressure (P_3) in terms of initial pressure (P_1) is

(a) $P_3 = 0.55\ P_1$ (b) $P_3 = 0.76\ P_1$ (c) $P_3 = 0.68\ P_1$ (d) $P_3 = 2.55\ P_1$

Ans. **[b]**

Sol.



In process BC

$$2P_1 \times \left(\frac{V}{2}\right)^{\gamma} = P_3(V)^{\gamma}$$

$$P_3 = 2P_1\left(\frac{1}{2}\right)^{\gamma}$$

$= 2 \times P_1(0.5)^{7/5}$

$= 2 \times P_1 \times 0.368$

$= 0.76\ P_1$

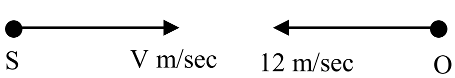
24. A car fitted with a device which transmits sound 60 times per minute. There is no wind and speed of sound in still air is 345 m/s. If you hear the sound 68 times per minute when you are moving towards the car with a speed of 12 m/s, the speed of the car must be nearly

(a) 20.0 m/s towards you (b) 30.0 m/s towards you

(c) 10.0 m/s away from you (d) 10.0 m/s towards you

Ans. **[b]**

Sol.



$$f' = f\left[\frac{v \pm v_0}{v \pm v_s}\right]$$

$$68 = 60\left[\frac{345+12}{345-v}\right]$$

v = 30 m/sec

25. A 43 m long rope of mass 5.0 kg joins two rock climbers. On climber strikes the rope and the second one feels the effect 1.4 s later. What is the tension in the rope ?

(a) 110 N (b) 301 N (c) 215 N (d) 154 N

Ans. **[a]**

Sol. $\mu = \frac{m}{\ell} = \frac{5}{43} = 0.116$

$v = \frac{\ell}{t} = \frac{43}{1.4} = 30.71 = \sqrt{\frac{T}{\mu}}$

$T = (30.71)^2 \times 0.116 = 110$ N

26. Two cats in a house mew at each other with sound intensities 5×10^{-9} Wm^{-2} and 9×10^{-6} Wm^{-2}. By how many decibels is the louder sound above the other ?

(a) ~13 dB (b) ~ 23 dB (c) ~ 33 dB (d) ~ 43 dB

Ans. **[c]**

Sol. $\beta = 10 \log\left(\frac{T_1}{T_0}\right)$

$\beta_1 = 10 \log \left(\frac{5\times10^{-9}}{10^{-12}}\right)$

$\Rightarrow 10 \log (5000)$

$\beta_1 = 10 \log(5 \times 10^3) = 10 \log (5 + 3)$

$\beta_2 = 10\log (9 + 6)$

$\beta_2 - \beta_1 = 10\{\log 9 - \log 5 + 3\}$

$= 2.5 + 30 = 32.5 \approx 33$

27. An electron orbiting around a nucleus has angular momentum L. The magnetic field produced by the electron at the centre of the orbit can be expressed as

(a) $B = (\mu_0 e / 8\pi mr^3)\, L$ (b) $B = (\mu_0 e / 4\pi mr^3)\, L$ (c) $B = (\mu_0 e / \pi mr^3)\, L$ (d) $B = (e / 4\pi\varepsilon_0 mr^3)\, L$

Ans. **[b]**

Sol. $B = \frac{\mu_0 i}{2r}$

$\frac{M}{L} = \frac{q}{2m}$

$\frac{i\times\pi r^2}{L} = \frac{e}{2m}$

$$i = \frac{eL}{2m\pi r^2}$$

$$B = \frac{\mu_0}{2r} \times \frac{eL}{2m\pi r^2} \Rightarrow \frac{\mu_0 eL}{4mr^3\pi}$$

28. The temperate at which average de Broglie wavelength of helium atom becomes 0.5 nm is

(a) 6.6 K (b) 7.1 K (c) 279.6K (d) 280.1 K

Ans. **[a]**

Sol. $$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}}{\sqrt{2m\times\frac{3}{2}KT}} = \frac{6.6\times10^{-34}}{\sqrt{4\times1.67\times10^{-27}\times3\times1.38\times10^{-23}}}$$

$$5 \times 10^{-10} = \frac{6.6\times10^{-34}}{\sqrt{26.5\times10^{-50}T}}$$

$$T = \left(\frac{6.6\times10^{-34}}{5\times10^{-10}}\right)^2 \times \frac{1}{26.5\times10^{-50}}$$

$T = 0.0656 \times 10^2$

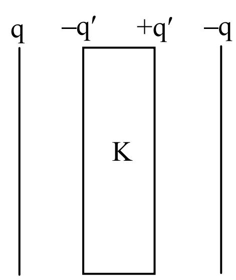
$T = 6.56$ K

29. A dielectric slab is introduced between the plates of a capacitor. If the charge on the capacitor is q and the magnitude of the induced charge on the dielectric surface is q′, then

(a) q′ < q (always) (b) q′ > q (always) (c) q′ = q (always) (d) q′ = 0

Ans. **[a]**

Sol.



$$q' = q\left[1 - \frac{1}{K}\right]$$

K > 1 for all dielectric

∴ q′ < q always

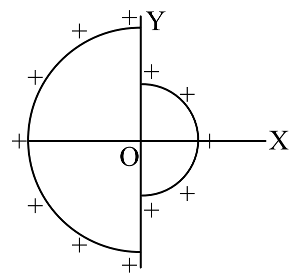
30. When two ends of a springs are pulled apart increasing its length it produces force equal to kx at its ends. At a point 1/4 of its length from one end the force is

(a) 0.25 kx (b) 0.75 kx (c) 1.0 kx (d) 0.5 kx

Ans. **[c]**

Sol. F = kx

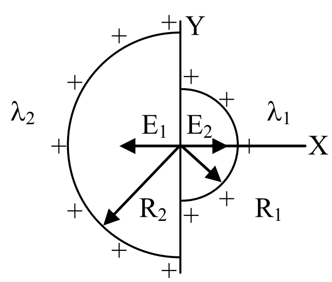
31. Two semicircular wires of radius 20 cm and 10 cm have a common centre at the origin O as shown in the figure. Assume that both the wires are uniformly charged and have an equal charge of 0.70 nC each. The magnitude of electric field at the common center of curvature O of the system is -



(a) 100 V/m (b) 301 V/m (c) 401 V/m (d) 501 V/m

Ans. **[b]**

Sol.



$$E = \frac{\lambda \sin \pi/2}{2\pi\varepsilon_0 R} \Rightarrow \frac{\lambda}{2\pi\varepsilon R}$$

$E_{net} = E_1 - E_2$

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 R} \Rightarrow \frac{q}{\pi R \times 2\pi\varepsilon R}$$

$$\Rightarrow \frac{q}{2\pi^2\varepsilon_0 R_1^2} \Rightarrow \frac{0.7\times10^{-9}}{2\times3.14^2\times8.85\times10^{-12}\times(0.1)^2}$$

$\Rightarrow 401.11$ V/m

$$E_2 = \frac{q}{2\pi^2\varepsilon_0 R_2^2} \Rightarrow \frac{0.7\times10^{-9}}{2\times3.14^2\times8.85\times10^{-12}\times(0.2)^2}$$

$= 100.2$

$E_{net} = E_1 - E_2 = 3.1$ V/m

32. An electron has its path unchanged when it is passing through a region of electric field ($E = 3.4 \times 10^4$ V/m) and a magnitude field ($B = 2 \times 10^{-3}$ Wb/m^2) both perpendicular to each other. However, on switching off the electric field, the electrons move along a circular path. What is the radius of circular path ?

(a) 4.82 m (b) 4.82×10^{-3} m (c) 4.82×10^{-2} m (d) none of the above

Ans. **[c]**

Sol. $v = \frac{E}{B}$

$v = \frac{3.4 \times 10^4}{2 \times 10^{-3}}$ $\Rightarrow 1.7 \times 10^7$ m/sec

$R = \frac{mV}{qB}$

$= v = \frac{9.1 \times 10^{-31} \times 1.7 \times 10^7}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} \Rightarrow 4.82 \times 10^{-2}$ m

33. A 1000 μF capacitor fully charged to 250 V discharges through a resistance wire embedded in a thermally insulated block of specific heat 2.5×10^2 J kg^{-1} K^{-1} and mass of 0.01 kg. How much is the increase in the temperature of the block ?

(a) 12.5 K (b) 8.5 K (c) 7.0 K (d) 15.5 K

Ans. **[a]**

Sol. Energy dissipated = $\frac{1}{2}CV^2$

$= \frac{1}{2} \times 1000 \times 10^{-6} \times (250)^2$

= 31.25 J

this energy increase the temperature of block

$31.25 = mS\Delta T$

$31.25 = 0.01 \times 2.5 \times 10^2 \, \Delta T$

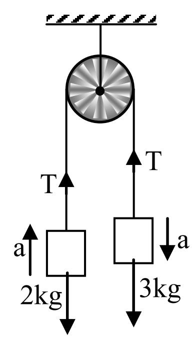
T = 12.5K

34. Two bodies A and B hanging in air are tied to the ends of a string which passes over a frictionless pulley. The mass of the string and the pulley are negligible and the masses of two bodies are 2 kg and 3 kg respectively. (Assuming g = 10 m/s^2). Body A moves upwards under a force equal to -

(a) 30 N (b) 24 N (c) 10 N (d) 4 N

Ans. **[d]**

Sol.



$a = \frac{3g - 2g}{3 + 2}$

a = 2 m/s^2

On 2 kg.

$3g - T = 3 \times \frac{g}{5}$

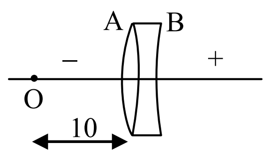
Fnet.= ma = 2 × 2 = 4N

35. An object is placed at a distance of 10 cm from a co-axial combination of two lenses A and B. The combination forms a real image three times the size of the object. If lens B is concave with a focal length 30 cm, then focal length of lens A is -

(a) 10 cm (b) 7.5 cm (c) 6 cm (d) – 6cm

Ans. **[c]**

Sol.



focal length of combination of lens = f_{eq}

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

m = – 3

$$\frac{v}{u} = -3$$

v = – 3u

$$\frac{1}{f_{eq}} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f_{eq}} = \frac{1}{-3u} - \frac{1}{u}$$

$$\frac{1}{f_{eq}} = \frac{-4}{3u}$$

$$\frac{1}{f_{eq}} = \frac{-4}{3(-10)}$$

$$\frac{1}{f_{eq}} = \frac{+30}{4}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{4}{30} = \frac{1}{-30} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{30} + \frac{4}{30}$$

$f_2 = 6$ cm

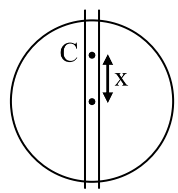
36. A body A revolves along a circular orbit close to the earth's surface. Body B oscillates along an imaginary straight tunnel drilled through the earth, whereas another body C through a similar longest tunnel. Let T_A, T_B and T_C be the corresponding periods of revolution of oscillation, then -

(a) $T_A > T_B = T_C$ (b) $T_A > T_C > T_B$ (c) $T_A = T_B = T_C$ (d) $T_A < T_B = T_C$

Ans. **[c]**

Sol.

For Body C



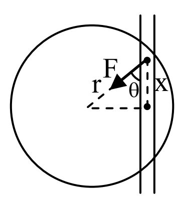
$$F = \frac{mGM}{R^3}x$$

$$a = \frac{GM}{R^3}x$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

$$T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{\sqrt{GM}}R^{3/2}$$

For Body B



acceleration along the tunnel $= \frac{F\cos\theta}{m}$

$$= \left(\frac{GM}{R^3} \times rm\cos\theta\right)\frac{1}{m}$$

$$= \left(\frac{GM}{R^3} \times rm\frac{x}{r}\right)\frac{1}{m}$$

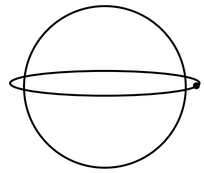
$$a = \frac{GM}{R^3}x$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

$$T = \frac{2\pi}{\omega}$$

$\therefore\ T_B = T_C$

For A



$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$v = \sqrt{\frac{GM}{R}}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi R^{3/2}}{\sqrt{GM}}$$

$\therefore\ T_A = T_B = T_C$

37. The vibrations of a string of length 100 cm and fixed at both ends are represented by the equation : $y = 2 \sin(\pi x / 10) \cos(50\pi t)$. Then the equations of the component waves whose superposition gives the above vibrations are -

(a) 2 sin $2\sin\left(\frac{\pi x}{10} + 50\pi t\right) + 2\sin\left(\frac{\pi x}{10} - 50\pi t\right)$

(b) $\sin\left(\frac{\pi x}{10} + 50\pi t\right) - \sin\left(\frac{\pi x}{10} - 50\pi t\right)$

(c) $\sin\left(\frac{\pi x}{10} + 50\pi t\right) + \sin\left(\frac{\pi x}{10} - 50\pi t\right)$

(d) $2\sin\left(\frac{\pi x}{10} + 50\pi t\right) + 2\sin\left(\frac{\pi x}{10} + 50\pi t\right)$

Ans. **[c]**

Sol. If we add two component

$$y = \sin\left(\frac{\pi x}{10} + 50\pi t\right) + \sin\left(\frac{\pi x}{10} - 50\pi t\right); \qquad y = 2\sin\left(\frac{\pi x}{10}\right)\cos(50\pi t)$$

38. Hot coffee in a mug cools from 90ºC to 70ºC in 4.8 minutes. the room temperature is 20ºC. Applying Newton's law of cooling the time needed to cool it further by 10ºC should be nearly -

(a) 4.2 min (b) 3.8 min (c) 3.2 min (d) 2.4 min

Ans. **[c]**

Sol. Average Newton cooling method

$$\frac{70-90}{4.8} = K\left[\frac{70+90}{2} - 20\right] \quad \ldots(1)$$

$$\Rightarrow -\frac{20}{4.8} = K[80-20]$$

$$\Rightarrow -\frac{20}{4.8} = K \times 60$$

for next cooling by 10ºC

$$\frac{60-70}{t} = K\left[\frac{60+70}{2} - 20\right] \quad \ldots(2)$$

$$\Rightarrow -\frac{10}{t} = K[65-20]$$

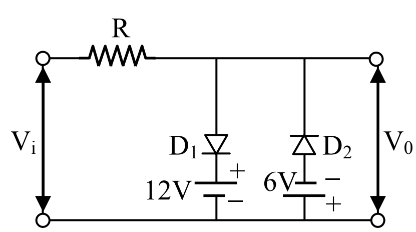
$$\Rightarrow -\frac{10}{t} = K \times 45$$

$$\Rightarrow -\frac{10}{t} = 45 \times \left(\frac{-20}{4.8 \times 60}\right)$$

$$t = \frac{+10 \times 3 \times 4.8}{45}$$

t = 3.2 minutes

39. A sinusoidal voltage of amplitude 15V is connected between the input terminals of the circuit shown in the figure. Assume that the diodes are ideal. In output waveform -



(a) positive peaks of the input will be clipped to +12 V and negative peaks will be clipped to – 6V

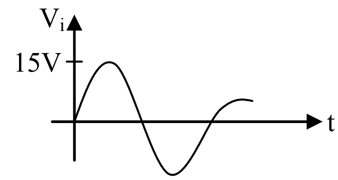
(b) positive peaks of the input will be clipped to +6 V and negative peaks will be clipped to – 12V

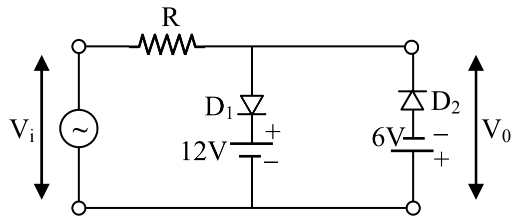
(c) positive peaks of the input will be clipped to +12 V and negative peaks will be clipped to – 12V

(d) positive peaks of the input will be clipped to +6 V and negative peaks will be clipped to – 6V

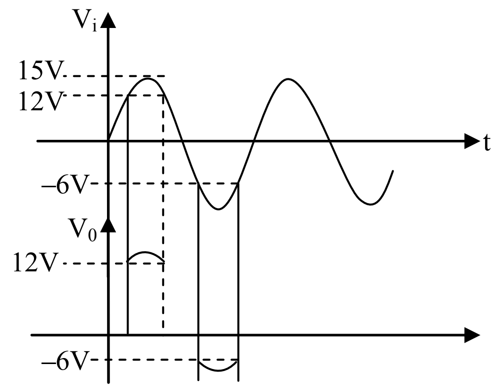
Ans. **[a]**

Sol.





When $v_i > 12$ V the $D_1 \rightarrow$ on and $D_2 \rightarrow$ off



40. Correlate a physicist among P, Q, R to an appropriate physicist among L, M, N. Let us pay a tribute to them

P. Louis de Broglie — L. Davison and Germer

Q. Max Plank — M. James Clerk Maxwell

R. Christian Huygen — N. Arthur H. Compton

(a) P & N, Q & M, R & L

(b) P & L, Q & N, R & M

(c) P & L, Q & M, R & N

(d) P & M, Q & L, R & N

Ans. **[b]**

SUB-PART A-2

In questions 41 to 50 any number of options (1 or 2 or 3 or all 4) may be correct. You are to identify all of them correctly to get 6 marks. Even if one answer identified is incorrect or one correct answer is missed, you get zero.

41. In a circuit carrying an alternating current -

(a) magnetic field around it oscillates with the frequency of the current

(b) heat is generated with double the frequency of the magnetic field

(c) voltage across the circuit is proportional to the rate of change of magnetic flux around it

(d) the current always lags in phase behind the applied voltage

Ans. **[a,b]**

42. The stress exerted by vehicles on the central part of a bridge with convex curvature will -

(a) be more than that at the ends

(b) same as that the ends

(c) be less than that at the ends

(d) decrease in proportion to the square of the speed of the vehicle

Ans. **[c,d]**

43. A cylindrical vessel filled with water is connected by a narrow pipe at its bottom to another identical empty vessel. Then -

(a) potential energy of water is proportional to the square of the height of its level

(b) 3/4th of the potential energy is lost when the water flow stops

(c) half the potential energy is lost when the levels in both the vessels are same

(d) loss in potential energy is equal to the rise in the thermal energy

Ans. **[a,c,d]**

44. The deviation produced by a prism depends upon -

(a) angle of incidence of prism (b) refracting angle of prism

(c) refractive index of prism (d) wavelength of light used

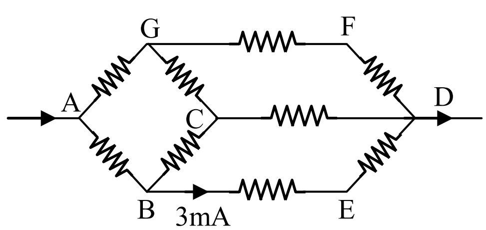
Ans. **[a,b,c,d]**

Sol. Deviation produced by prism

$D = i + e - A$

e depend on refractive index of prism

45. Consider nine identical resistances arranged as shown in the figure. In this arrangement electric current enters at anode A and leaves from node D. Let $V_{AD} = 5$ volt and $I_{BE} = 3$ mA, therefore -



(a) $I_{AB} = 5$ mA

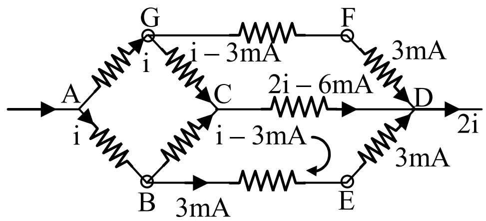
(b) each resistance is (5000/11) Ω

(c) effective resistance between A and D is 500 Ω

(d) power dissipation along the path BCD is (100/11) mW

Ans. **[a,b,c,d]**

Sol.



potential of G and B is same

potential of F and E is same

KVL in CDEB

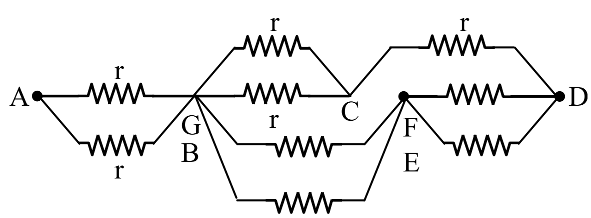
$(2i - 6 \times 10^{-3}) \times R - 6 \times 10^{-3} R + (1 - 3 \times 10^{-3}) R = 0$

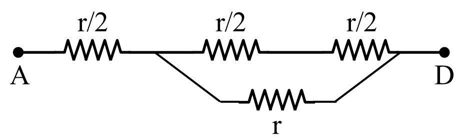
$3i = 15 \times 10^{-3}$

$i = 5 \times 10^{-3}$ A

current in AB = 5 mA

equivalent circuit





$R_{eq} \Rightarrow \frac{r}{2} + \frac{3r}{5} \Rightarrow \frac{11r}{10}$

$R_{eq} = \frac{V}{2i} \Rightarrow \frac{5}{2 \times 5 \times 10^{-3}} \Rightarrow 500\ \Omega$ **Ans. (c)**

$500 = \frac{11r}{10}$

$r = \frac{5000}{11}$ **Ans. (b)**

Power in BCD $\Rightarrow (1 - 3)^2 \times r + (21 - 6)^2 \times r$

$= 4 \times 10^{-6} \times r + 16 \times 10^{-6} \times r$

$= 20 \times 10^{-6} \times r$

$= 2 \times 10^{-5} \times \frac{5000}{11}$

$= \frac{100}{11}$ m watt

46. Mark the correct statement(s) of the following -

(a) A convex mirror forms virtual images for all the positions of object

(b) A convex mirror forms real images for all the positions of object

(c) A convex mirror can form a virtual magnified image

(d) The magnification produced by a convex mirror is always less than unity

Ans. **[a,c,d]**

Sol. (d) option is correct if object is real.

(a) is correct if object is real.

So, if object is real then ans. a, c, d or

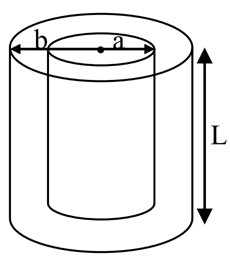
if object is not real then ans. (c)

47. A solid cylindrical conductor of radius a and charge q is coaxial with a cylindrical shell of negligible thickness, radius b (>a) and charge –q. The capacitance of this cylindrical capacitor per unit length is proportional to -

(a) log (b/a) (b) log (a/b) (c) $[\log (b/a)]^{-1}$ (d) 1/[log b – log a]

Ans. **[c,d]**

Sol.



$$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}$$

$$\frac{C}{L} \propto \left(\ln\frac{b}{a}\right)^{-1} \qquad \left[\log\frac{a}{b} = \frac{1}{\ln\frac{b}{a}}; \log x = \log\frac{1}{x}\right]$$

$$\frac{C}{L} \propto \left(\frac{1}{\ln b - \ln a}\right)$$

48. Mark the correct statement(s) of the following -

(a) In case of liquids the boiling point increases with pressure for all liquids

(b) In case of solid the melting point decreases with pressure for all solids

(c) In case of ice the melting point decreases with pressure

(d) In case of ice the melting point increases with pressure

Ans. **[a,c]**

49. The electric potential (in volt) in a region along the x-axis varies with x according to the relation $V(x) = 5 + 4x^2$, where x is in m. Therefore

(a) the potential difference between the points x = 1 and x = – 3 is 32 V

(b) force experienced by a charge of 1 C placed at x = – 1 m is 8 N

(c) force experienced by the above mentioned charge is along the +ve x-axis

(d) a uniform electric field exists in this region along the x-axis

Ans. **[a,b,c]**

Sol. at x = 1

$V_1 = 5 + 4 \times 1^2 \Rightarrow 9V$

at x = – 3

$V_3 = 5 + 4 \times 3^2 \Rightarrow 41V$

$V_1 - V_3 = 9 - 41$

$= -32$ V

$E = -\frac{dv}{dx}\hat{i}$

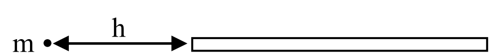
$= -8x\hat{i}$

at x = – 1m, F = qE

$= 1 \times (-8 \times (-1))\hat{i} \Rightarrow 8\hat{i}$

electric field is non uniform.

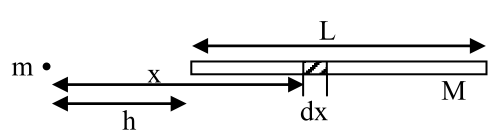
50 A homogeneous bar of length L and mass M is situated at a distance h from a particle of mass m as shown. The gravitational force exerted by the bar on the particle varies inversely as -



(a) $(L - h)^2$ (b) $(h + L/2)^2$ (c) $h(h + L)$ (d) h^2 if $L << h$

Ans. **[c,d]**

Sol.



Force on element $dF = \frac{Gm \times \lambda dx}{x^2}$

Net force on bar $= \int_{x=h}^{x=L+h} \frac{GM}{x^2} \lambda dx$

$= GM\lambda \left[-\frac{1}{x}\right]_h^{L+h}$

$= -\frac{GMM}{L}\left[\frac{1}{L+h} - \frac{1}{h}\right]$

$F = +GM^2 \left[\frac{L}{(L+h)(h)}\right]$

If L <<<< h

$F \propto \frac{1}{h^2}$

PART- B — Marks – 60

All the questions are compulsory. **All questions carry equal marks.**

1. A particle is moving in positive x direction with its velocity varying as $v = \alpha\sqrt{x}$. Assume that at t = 0 the particle was located at x = 0. Determine (i) the time dependence of velocity (ii) acceleration and (iii) the mean velocity of the particle average over the time that the particle takes to cover the fires s meters of the path.

Sol. $v = \alpha\sqrt{x}$

$\frac{dx}{dt} = \alpha\sqrt{x}$

$x^{-1/2}\, dx = \alpha\, dt$

$2\, x^{1/2} = \alpha t$

$x = \frac{\alpha^2 t^2}{4}$

$v = \frac{\alpha^2}{4}(2t)$

$v = \frac{\alpha^2 t}{2}$ **Ans.**

$a = \frac{\alpha^2}{2}$

$$\frac{ds}{dt} = \frac{\alpha^2 t}{2}$$

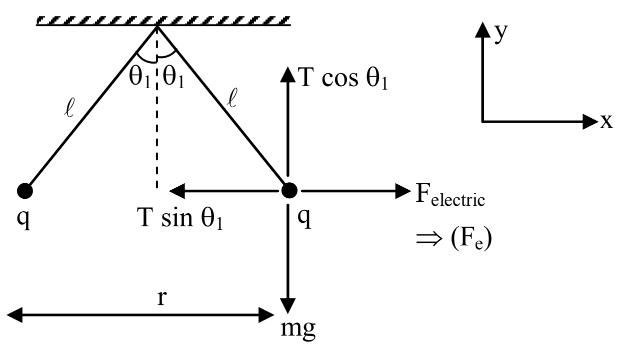
$$s = \frac{t^2\alpha^2}{4}$$

$v_{av} = s/t$

$$v_{av} = \frac{\alpha^2 t}{4}$$ **Ans.**

2. Two identical metal spheres of density ρ having equal and similar charges are supported from a common point by means of a silk thread of length ℓ and negligible mass. The two thread make an angle $2\theta_1$ with each other when in equilibrium in air. When the same system is immersed in a dielectric liquid of density σ, then the angular separation changes to $2\theta_2$. Then find (i) the relative permittivity (ε_r) of the liquid in terms of ρ, σ, θ_1, θ_2 (ii) In case the angular separation remains unchanged even on immersing the system in the dielectric liquid find the expression for ε_r.

Sol.



$\Sigma F_x = 0$

$T \sin\theta_1 = F_e$

$$T \sin\theta_1 = \frac{kq^2}{r^2}$$

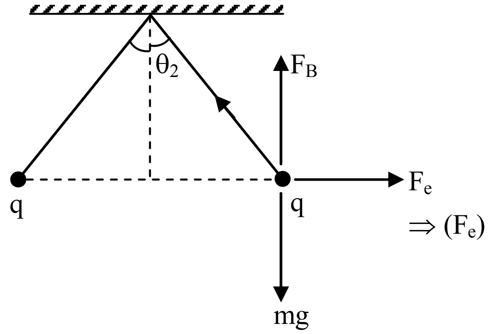
$T \cos\theta_1 = mg$

$T \cos\theta_1 = V\rho g$

$$\tan\theta_1 = \frac{kq^2}{r^2 V\rho g} \qquad \ldots(1)$$

Here $r = 2\ell \sin\theta_1$

when system is immersed in liquid of density σ. Then $F_e = \dfrac{kq^2}{r^2 k}$ where $k = \dfrac{\varepsilon}{\varepsilon_r}$



and $r = 2\ell \sin \theta_2$

$T \cos \theta_2 + F_B = mg$

$T \cos \theta_2 = mg - V\sigma g$

$T \cos \theta_2 = Vg[\rho - \sigma]$

$T \sin \theta_2 = \dfrac{kq^2}{\varepsilon^2 k}$

$\tan \theta_2 = \dfrac{kq^2}{r^2 k}\ Vg\,[\rho - \sigma]$

$\tan \theta_2 = \dfrac{kq^2}{4\ell^2 \sin^2 \theta_2 Vg[\rho - \sigma]k}$

from (i)

$\tan\theta_2 = \dfrac{\tan\theta_1 \times V\rho g \times 4\ell^2 \sin^2\theta_1}{4\ell^2 \sin^2\theta_2 \times g[\rho - \sigma]k}$

$\dfrac{\varepsilon}{\varepsilon_r} = k = \dfrac{\tan\theta_1 \times \sin^2\theta_1}{\tan\theta_2 \times \sin^2\theta_2} \times \dfrac{\rho}{\rho - \sigma}$ **Ans.**

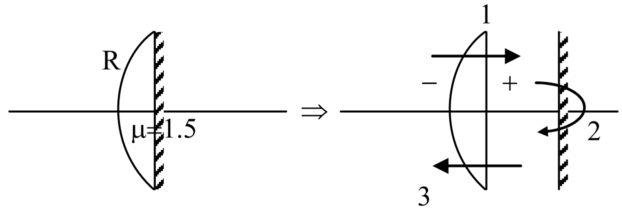
(ii) if angular separation remain unchanged i.e. $\theta_1 = \theta_2$

then $k = \dfrac{\rho}{\rho - \sigma}$ **Ans.**

3. (a) The plane side of a thin planoconvex lens is silvered so that the lens acts as a concave mirror of focal length 40 cm. The material of lens refractive index 1.5. Determine the radius of curvature of the curved surface of the lens.

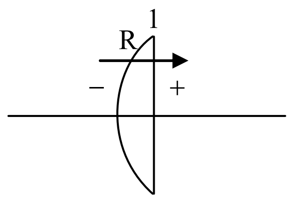
(b) Light falls on one end of a cylindrical glass rod at an angle α. Determine the smallest refractive index that the glass may have so that the light after entering the rod does not leave it through its curved surface irrespective of the value of α.

Sol. (a)



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

This whole system go through three events f_{eq} is the focal length of equivalent mirror



$$\frac{1}{f_1} = \frac{1}{f_3} = \frac{1.5-1}{1}\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$\frac{1}{f_3} = \frac{1}{f_1} \Rightarrow \frac{0.5}{R}$$

$f_2 = \infty$ [focal length of plane mirror = ∞]

$$\frac{1}{f_{eq}} = \frac{2}{f_1} + \frac{1}{\infty}$$

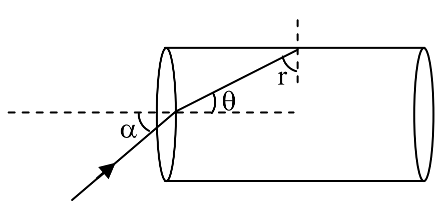
$$\frac{1}{f_{eq}} = 2 \times \frac{0.5}{R}$$

$f_{eq} = 40$ (given)

$$\frac{1}{40} = \frac{1}{R}$$

R = 40 cm **Ans.**

(b)



$r + \theta = 90^o$

$r = 90 - \theta$

for different value of α, θ will have different value

$\therefore$ r will have different value.

light should not go out of curved surface

$\therefore$ r must be greater than θ_C

$r > \theta_C$

as r has various value

$\therefore r_{min} > \theta_C$ so

that all r can be θ_C

$r_{min} = 90 - \theta_{max}$

θ is θ_{max} when $\alpha = 90$

$\theta_{max} = \theta_C$

$r_{min} = 90 - \theta_C$

$45 > \theta_C$

$\frac{1}{\sqrt{2}} > \frac{1}{\mu}$

$\mu > \sqrt{2}$ **Ans.**

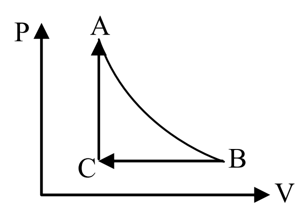
4. A cyclic process is indicated in the following PV diagram. In the initial state (A) temperature, pressure and volume of the system are 300 K, 1 atm and 1000 cc. In the first process (AB), the adiabatic expansion increases the volume to 2000 cc. This is followed by an isobaric compression to reduce the volume of gas to 1000cc. The gas is brought to initial state by isochoric process. The system is 1 mole of a monatomic gas.

a) Find the pressure and temperature at state B and C

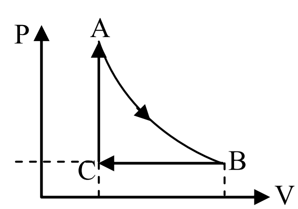
b) Calculate the work done in adiabatic, isobaric, isochoric processes.

c) Calculate the total work done in cyclic process and amount of energy supplied to the system

d) Calculate the efficiency of cyclic process.



Sol.



At A $T_A = 300$ K, $P_A = 1$ atm, $V_A = 1000$ cm^3

AB adiabatic process

$V_B = 2000\ cm^3$

$\gamma = 1 + \frac{2}{f}$ and $1 + \frac{2}{3} = \frac{5}{2}$

$PV^\gamma = C$

$P_A \times V_B^{5/2} = P_A V_A^{5/2}$

(a) $P_B = \left(\frac{V_A}{V_B}\right)^{5/2} \times P_A = \left(\frac{1}{2}\right)^{5/2} \times 1$

= 0.2 atm **Ans.**

$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$

$T_B = T_A \times \left(\frac{V_A}{V_B}\right)^{\frac{5}{2}-1}$

$\Rightarrow 300 \left(\frac{1}{2}\right)^{3/2} = 300 \times 0.33$

= approx 100ºK **Ans.**

(b) $W_{AB} = -\Delta U_{AB}$

$= -\frac{f}{2}\ nR\ [T_B - T_A] \Rightarrow -\frac{3}{2} \times 1 \times R\ [300 \times 0.33 - 300]$

Work done in adiabatic process = + 2505.47 Joule **Ans.**

W_{BC} = Work done in isobaric process = $nR[T_C - T_B]$

$T_B = 300 \times 0.33$

$V_B = 2000\ cm^3$

$V_C = V_A = 1000\ cm^3$

Isobaric process $\frac{V}{T} = C$

$V \propto T$

$\therefore$ as volume get half temperature also get half.

$T_C = \frac{T_B}{2}$

$W_{BC} = 1 \times 8.31 \times \left[\frac{T_B}{2} - T_B\right]$

$= -\frac{8.31 \times T_B}{2}$

= – 411.34 J **Ans.**

$W_{CA} = 0$ as volume is constant

(d) Net work done = $W_{AB} + W_{BC} + W_{CA}$

$= 2505.47 - 411.34 + 0$

$= 2094.13$ J **Ans.**

$Q_{heat\ supply} = Q_{CA} = n_{cv}(T_A - T_C)$

$$\Rightarrow 1 \times \frac{3}{2} \times 8.314 \times \left(300 - \frac{300 \times 0.33}{2}\right)$$

$= 3123.98$ J

efficiency of cycle $= \frac{W_{total}}{Q_{heat}} = \frac{2094.13}{3123.98}$

$= 67\%$ or 0.67 **Ans.**

5. A 750 Hz, 20 V source is connected to a resistance of 100 Ω, a capacitances of 1.0 μF and an inductance of 0.18 H in series. Calculate the following quantities :

(a) Impedance of the circuit

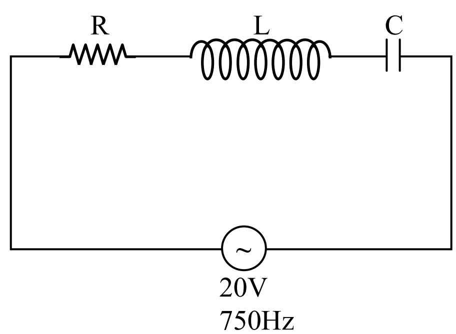
(b) Draw an impedance diagram with a suitable scale.

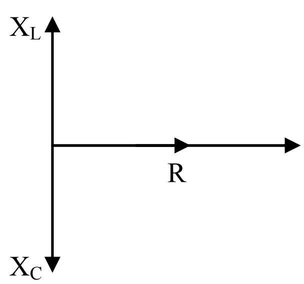
(c) Power factor

(d) The time in which the resistance will get heated by 10º C, provided that the thermal capacity of resistance = 2J/ºC.

Sol. (a) Impedance $z = \sqrt{(X_L - X_C)^2 + R^2} = 643.32\ \Omega$ **Ans.**

(b)





$X_L = \omega L = 2\pi \times 750 \times 0.18 = 847.8\Omega$

$$X_C = \frac{1}{2\pi \times 750 \times 1 \times 10^{-6}} = \frac{10^6}{2\pi \times 750} = 212.3\Omega$$

$R = 100\ \Omega$ **Ans.**

(c) Power factor = $\cos\phi = \frac{R}{Z} = \frac{100}{643.32} = 0.155$ **Ans.**

(d) Heat required = ms ΔT, $2 \times 10 = 20$ J

Average power = $i^2_{rms} \times R = \left(\frac{V}{2}\right)^2 \times R$

$= \left(\frac{20}{643.32}\right)^2 \times 100$ watt

= 0.096 watt

t is the time to receive energy of 20 Joule

$0.096 \times t = 20$

t = 206.9 sec **Ans.**

*****

Physical constant you may need ....

1. Charge on electron $e = 1.6 \times 10^{-19}$C
2. Mass of electron $m_e = 9.1 \times 10^{-31}$ kg
3. Universal gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2
4. Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2
5. Gas constant R = 8.31 J/K mol
6. Planck constant $h = 6.62 \times 10^{-34}$ Js
7. Stefan constant $\sigma = 5.67 \times 10^{-8}$ $W/m^2 K^4$
8. Boltzman constant $k = 1.38 \times 10^{-23}$ J/K
9. Mass of proton $m_p = 1.67 \times 10^{-27}$ kg