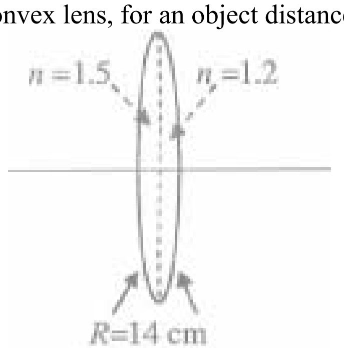
CODE : 9 **08 / 04 / 2012**

Part – I : (PHYSICS)

SECTION – I (Single Correct Answer Type)

This section contains 10 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Q.1 A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be –



(A) – 280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Ans. **[B]**

Sol. For the combination $\frac{1}{f_{eq}} = \frac{(\mu_1 - 1)}{R} + \frac{(\mu_2 - 1)}{R}$

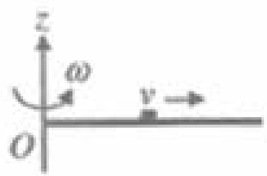
$\boxed{f_{eq.} = 20}$

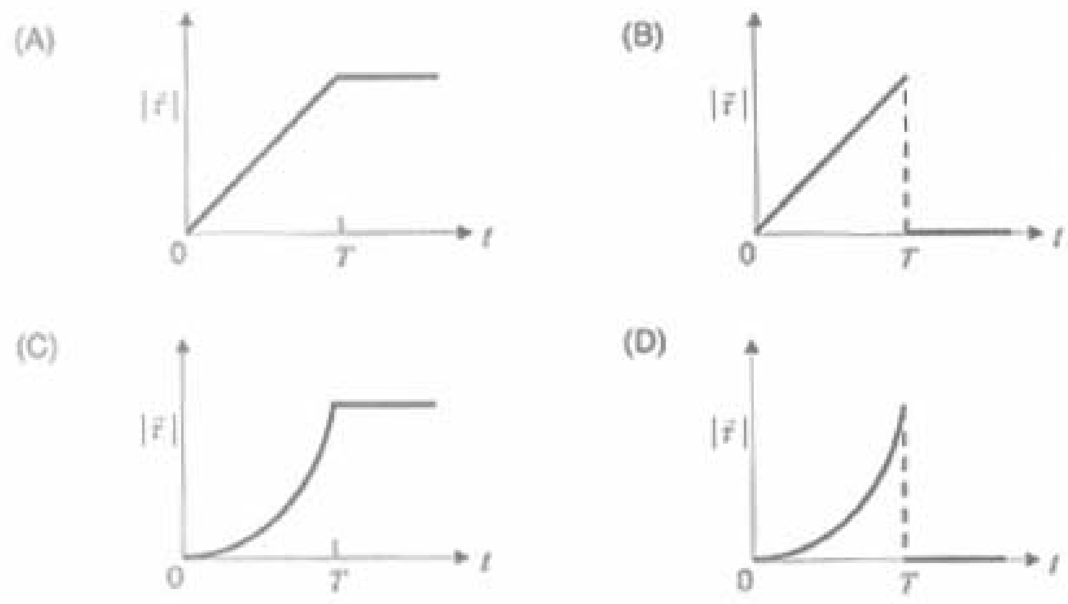
Here u = – 40, f = 20

$\boxed{v = 40}$

Q.2

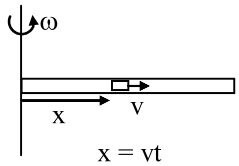
A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) on the system about O, as a function of time is best represented by which plot ?





Ans. [B]

Sol. Angular momentum about rotational axis



x = vt

$L_{(t)} = [I + m(vt)^2]\omega$

$\frac{dL_t}{dt} = 2mv^2t\omega$;

torque $\tau = (2mv^2\omega)t$

Q.3

Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$ (C) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (D) $(97)^{\frac{1}{4}} T$

Ans. [C]

Sol. Under steady state $\sigma A[(2T)^4 - T_1^4] = \sigma A[T_1^4 - (3T)^4]$

$(2T)^4 - T_1^4 = T_1^4 - 3^4T^4$

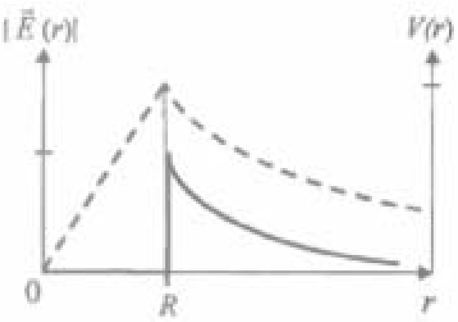
$2T_1^4 = (2^4 + 3^4)T^4$; $2T_1^4 = (16 + 81)T^4$

$T_1 = \left(\frac{97}{2}\right)^{1/4} T$

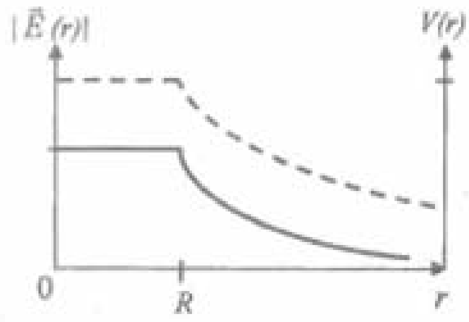
Q.4

Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph ?

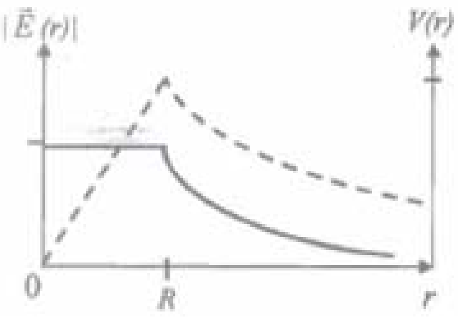
(A)



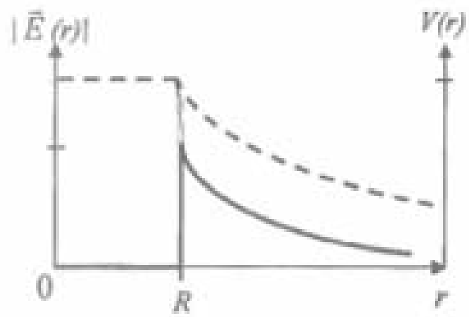
(B)



(C)

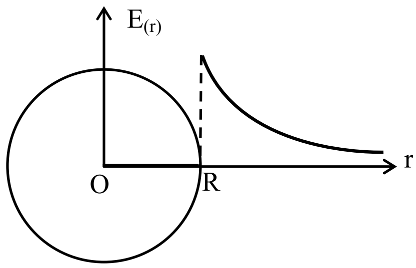


(D)



Ans. **[D]**

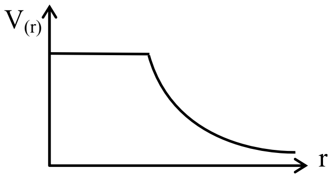
Sol.



$r < R$; $E_r = 0$

$r > R$; $E_r \propto 1/r^2$

$r < R$, $V = \dfrac{KQ}{R}$; $r = R$, $V = \dfrac{KQ}{R}$; $r > R$, $V = \dfrac{KQ}{R}$



Q.5

In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and l are the same.

(B) due to the error in the measurement of d is twice that due to the error in the measurement of l.

(C) due to the error in the measurement of l is twice that due to the error in the measurement of d.

(D) due to the error in the measurement of d is four times that due to the error in the measurement of l.

Ans. **[A]**

Sol. $Y = \frac{4MLg}{\pi \ell d^2}$ & % y_{max} = %M + %L + %ℓ + 2%d

Least count of both instrument, $\Delta\ell = \Delta d = \frac{0.5}{100} = 5\times10^{-3}$

$\%\ell = \frac{\Delta\ell}{\ell}\times100 = \frac{5\times10^{-3}}{0.25} = 2\%$

$\%d = \frac{\Delta d}{d}\times100 = \frac{5\times10^{-3}}{0.5}\times100 = 1\%$

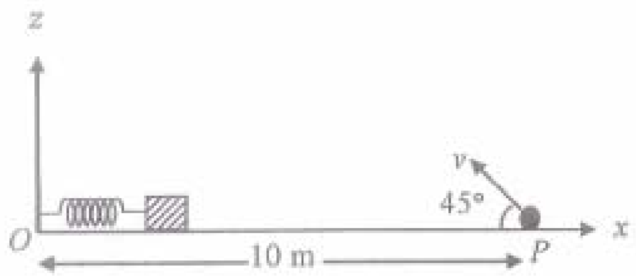
here we see that, contribution of ℓ, = 2%

contribution of d = 2% d = 2 × 1 = 2%

hence both terms ℓ and d contribute equally.

Q.6

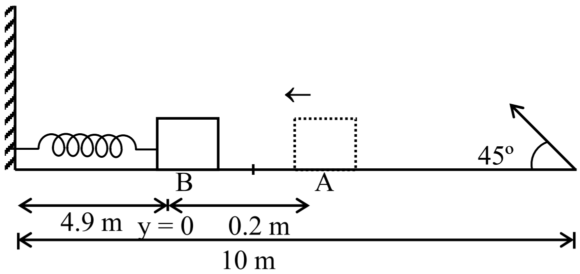
A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10\ m/s^2$)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Ans. **[A]**

Sol.



The block is released from A.

$x = 4.9\text{ m} + (0.2\text{ m})\sin\left(\omega t + \frac{\pi}{2}\right)$

at t = 15 ; x = 5 m

So range of projectile will be 5 m

Now $5 = \frac{v^2 \sin 90°}{g}$

$\Rightarrow \quad v^2 = 50$

$\Rightarrow \quad v = \sqrt{50}$

Q.7

Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

Ans. **[D]**

Sol. As, $\beta = \frac{\lambda D}{d}$

$\lambda_R > \lambda_G > \lambda_B$

So $\beta_R > \beta_G > \beta_B$

Q.8

A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then

(A) $\vec{L}_O$ and $\vec{L}_P$ **do not** vary with time.

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

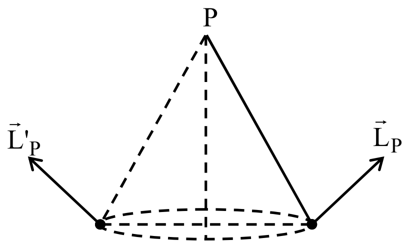
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

Ans. **[C]**

Sol. $\vec{L}_0 = \vec{r}_0 \times \vec{p}$

$\vec{L}_0$ is always directed along the axis & its magnitude is constant.



Q.9

A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Ans. [D]

Sol. $v_{rms} = \sqrt{\frac{3KT}{m}}$

$$\frac{v_{rms(\text{He})}}{v_{rms(\text{Ar})}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

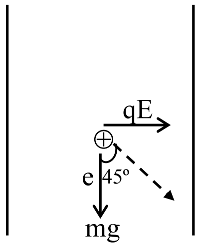
Q.10

Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45^0 to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

Ans. [C]

Sol.



qE = mg

$$E = \frac{mg}{q}$$

$$\frac{V}{1\times10^{-2}} = \frac{mg}{q}$$

$$V = \frac{10^{-2}\times9.1\times10^{-31}\times1830\times100}{1.6\times10^{-19}}$$

$= 10^{-12} \times 9.1 \times 114$

$= 1000 \times 10^{-12}$

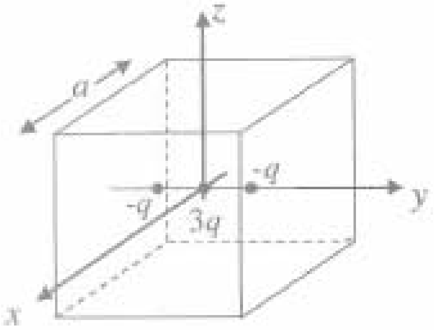
$V = 1 \times 10^{-9}$ V

SECTION – II (Multiple Correct Answer(s) Type)

This section contains 5 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE** are correct.

Q.11

A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0,0,0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s).



(A) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$.

(B) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(D) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

Ans. [A, C, D]

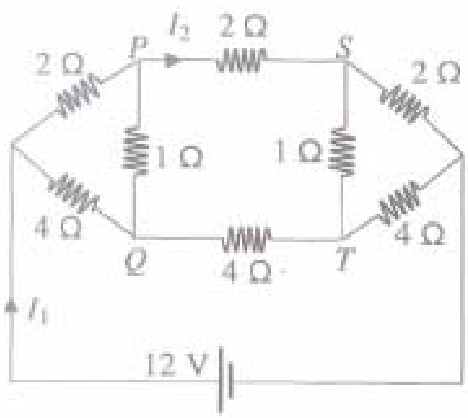
Sol. $\phi_{out} = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

By symmetry

$\therefore$ Ans is (A, C, D)

Q.12

For the resistance network shown in the figure, choose the correct option(s).

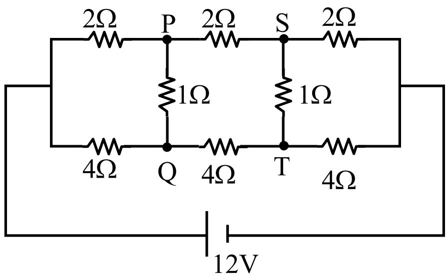


(A) The current through PQ is zero. (B) $I_1 = 3$ A.

(C) The potential at S is less than that at Q. (D) $I_2 = 2$ A.

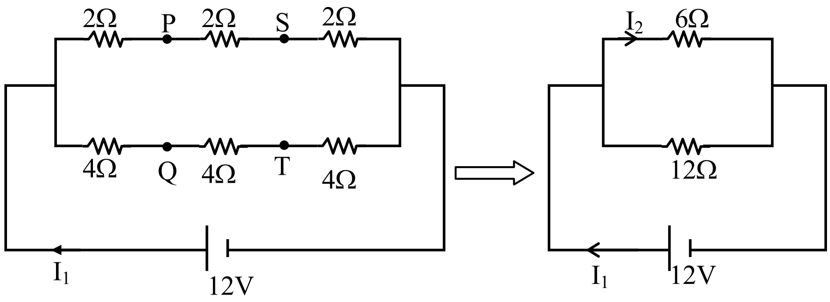
Ans. **[A,B,C,D]**

Sol. The circuit can be simplified as



because of symmetry no current passes through 1Ω resistor.

⇓



$$R_{eq} = \frac{12\times 6}{12+6} = 4\Omega$$

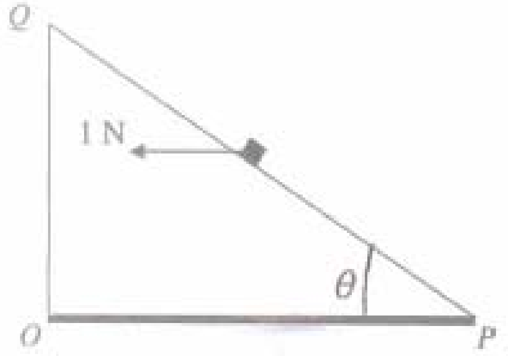
$I_1 = \frac{12}{4} = 3A$; $I_2 = \frac{2}{3}\times 3 = 2A$

Here we have

$\boxed{V_S - V_Q = -4}$ i.e., $\boxed{V_S < V_Q}$

Q.13

A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$)



(A) $\theta = 45^o$.

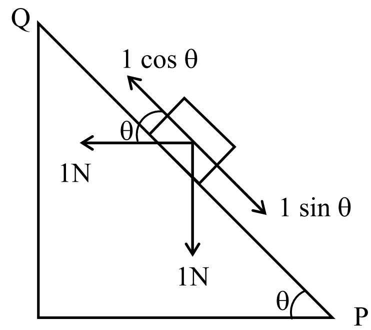
(B) $\theta > 45^o$ and a frictional force acts on the block towards P.

(C) $\theta > 45^o$ and a frictional force acts on the block towards Q.

(D) $\theta < 45^o$ and a frictional force acts on the block towards Q.

Ans. **[A, C]**

Sol.



If θ = 45º then cos θ = sin θ hence block will be at rest.

If plane is rough & θ > 45° then sin θ > cos θ so friction will act up the plane

If plane is rough & θ < 45 then cos θ > sin so friction will act down the plane so (A, C) are correct

Q.14

Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\,\hat{j}$ and $\vec{B} = B_0\,\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?

(A) If $\theta = 0^\circ$, the charge moves in a circular path in the x-z plane.

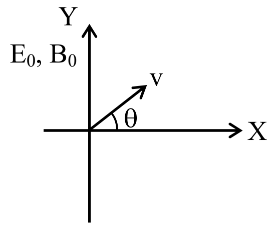
(B) If $\theta = 0^\circ$, the charge undergoes helical motion with constant pitch along the y-axis.

(C) If $\theta = 10^\circ$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(D) If $\theta = 90^\circ$, the charge undergoes linear but accelerated motion along the y-axis.

Ans. **[C, D]**

Sol.



* If θ = 0 or 10°
  then particle moves in helical path with increasing pitch along Y-axis.
* If θ = 90° then magnetic force on the particle is zero and particle moves along Y-axis with constant acceleration.

Q.15 A person blows into open end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(A) a high pressure pulse starts traveling up the pipe, if the other end of the pipe is open.

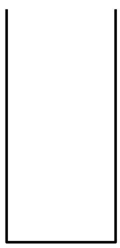
(B) a low pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(C) a low pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

(B) a high pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

Ans. **[B, D]**

Sol.



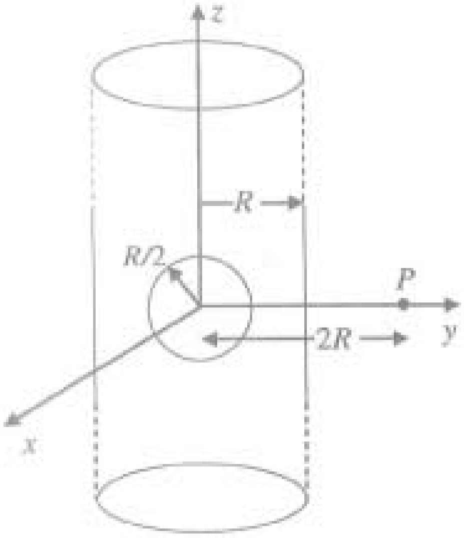
At rigid end there is no phase difference in pressure wave, when end is open phase difference of π come in pressure wave.

SECTION – III (Integer Answer Type)

This section contains 5 **questions**. The answer to each question is a **SINGLE DIGIT INTEGER**, ranging from 0 to 9 (both inclusive).

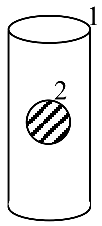
Q.16

An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



Ans. **[6]**

Sol.



(1) + (2) = Complete cylinder

$E_1 + E_2 = E$

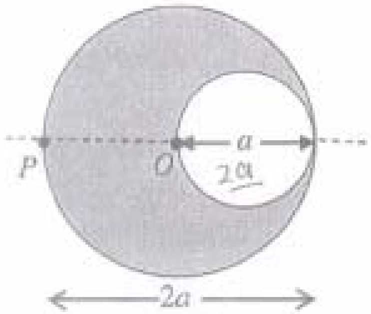
$$E = \frac{\rho \times \pi R^2}{2\pi \in_0 (2R)} = \frac{\rho R}{4 \in_0}$$

$$E_2 = \rho \times \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 \times \frac{1}{4\pi \in_0 (4R^2)} = \frac{\rho R}{24 \times 4 \in_0}$$

$$E_1 = E - E_2 \Rightarrow \frac{\rho R}{4 \in_0}\left[1 - \frac{1}{24}\right] = \frac{\rho R}{4 \in_0}\,\frac{23}{4 \times 6} = \frac{23\rho R}{16 \in_0 \times 6}$$

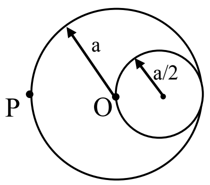
Q.17

A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Ans. [5]

Sol.



$$I = J \times \pi a^2$$

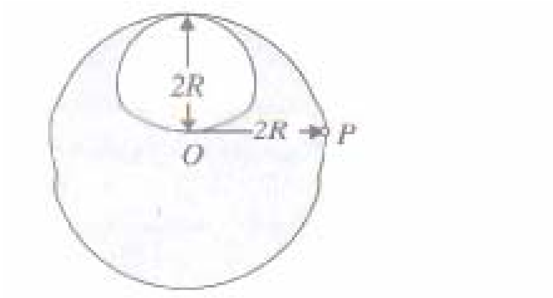
$$B = \frac{\mu_0 J \times \pi a^2}{2\pi a} - \frac{\mu_0 J \times \pi}{2\pi \times \frac{3a}{2}} \times \frac{a^2}{4}$$

$$B = \mu_0 Ja \left[\frac{1}{2} - \frac{1}{12}\right]$$

$$B = \mu_0 Ja \times \frac{5}{12}$$

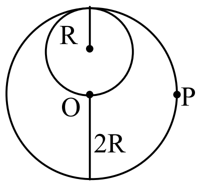
Q.18

A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



Ans. [3]

Sol.



Let mass of original disc = m

The mass of disc removed = $\frac{m}{\pi(4R^2)} \times \pi R^2 = \frac{m}{4}$

So M.O.I of remaining section about axis passing

through "O" $I_0 = \frac{m(2R)^2}{2} - \left[\frac{m}{4}\frac{R^2}{(2)} + \frac{m}{4}R^2\right]$

$\Rightarrow 2mR^2 - \left[\frac{mR^2 + 2mR^2}{8}\right] \Rightarrow \left[2 - \frac{3}{8}\right]mR^2 \Rightarrow \frac{13}{8}mR^2$

MOI of remaining section about "P"

$$I_P = \left[\frac{m(2R)^2}{2} + m(2R)^2\right] - \left[\frac{m}{4}\frac{R^2}{(2)} + \frac{m}{4}5R^2\right]$$

$$\Rightarrow [2mR^2 + 4mR^2] - \left[\frac{mR^2}{8} + \frac{5mR^2}{4}\right]$$

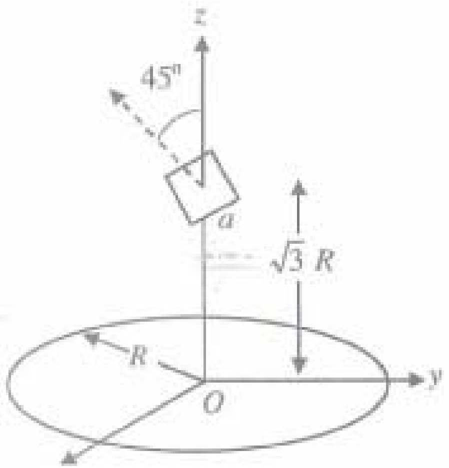
$$\Rightarrow 6mR^2 - \frac{11}{8}mR^2$$

$$\Rightarrow \frac{37}{8}mR^2$$

$$\frac{I_P}{I_O} = \frac{37}{8} \times \frac{8}{13} \approx 3$$

Q.19

A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is



Ans. [7]

Sol. Assume circular wire loop as primary and square loop as secondary coil

$$\phi_{secondary} = \frac{2\mu_0 i R^2}{2(3R^2 + R^2)^{3/2}} \times a^2 \times \cos 45^o$$

$$= \frac{\mu_0 i R^2}{2 \times 8R^3} \times a^2 \times \frac{2}{\sqrt{2}}$$

$$M = \frac{\phi_{secondary}}{i} = \frac{\mu_0 a^2}{2^3 \times 2^{1/2} R}$$

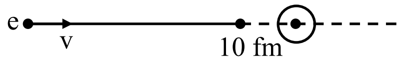
$$M = \frac{\mu_0 a^2}{2^{\frac{7}{2}} R}$$

Q.20

A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m)

Ans. **[7]**

Sol.



$U_i = 0$

$U_f = \dfrac{k \times 120e^2}{10 \times 10^{-15}}$

$K_i = \dfrac{1}{2}mv^2$ and $K_f = 0$

From Energy conservation,

$K_f + U_f = K_i + U_i$

$\dfrac{K \times 120e^2}{10 \times 10^{-15}} = \dfrac{1}{2}mv^2$

$v = 5.76 \times 10^7$

$\lambda = \dfrac{h}{mv} = \dfrac{6.67 \times 10^{-34}}{\frac{5}{3} \times 10^{-27} \times 5.76 \times 10^7}$

= 7 fm (approx)

Part – II : (CHEMISTRY)

SECTION – I SINGLE CORRECT ANSWER TYPE

This section contains 10 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Q.21 As per IUPAC nomenclature, the name of the complex $[Co(H_2O)_4(NH_3)_2]Cl_3$ is -
(A) Tetraaquadiaminecobalt (III) chloride
(B) Tetraaquadiamminecobalt (III) chloride
(C) Diaminetetraaquacobalt (III) chloride
(D) Diamminetetraaquacobalt (III) chloride

Ans. **[D]**

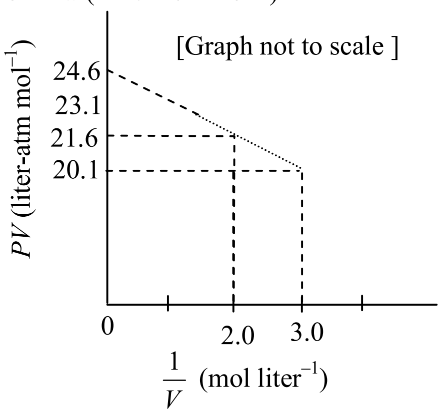
Sol. Factual

Q.22 In allene (C_3H_4), the type (s) of hybridization of the carbon atoms is (are)
(A) sp and sp^3 (B) sp and sp^2 (C) only sp^2 (D) sp^2 and sp^3

Ans. **[B]**

Sol. $CH_2=C=CH_2$
$\downarrow \quad \downarrow \quad \downarrow$
$sp^2 \quad sp \quad sp^2$

Q.23 For one mole of a van der Waals gas when $b = 0$ and $T = 300$ K, the PV vs. $1/V$ plot is shown below. The value of the van der Waals constant a (atm. liter^2 mol^{-2})



(A) 1.0 (B) 4.5 (C) 1.5 (D) 3.0

Ans. **[C]**

Sol. $\left(P + \frac{a}{V^2}\right) \times V = RT$

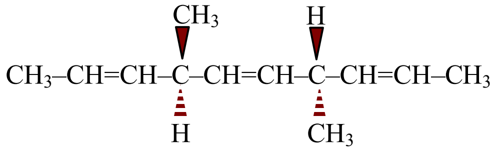
$PV = RT - \frac{a}{V}$

Slope = – a

$= \frac{20.1 - 21.6}{1} = -1.5$

$\Rightarrow a = 1.5$

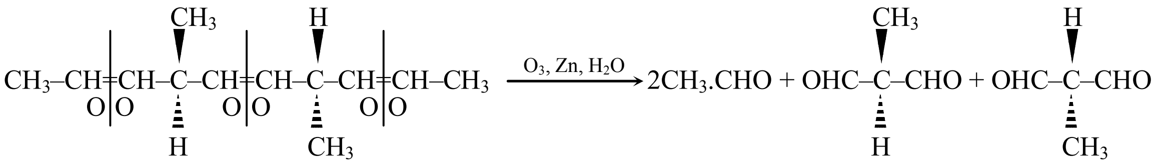
Q.24 The number of optically active products obtained from the **complete** ozonolysis of the given compound is –



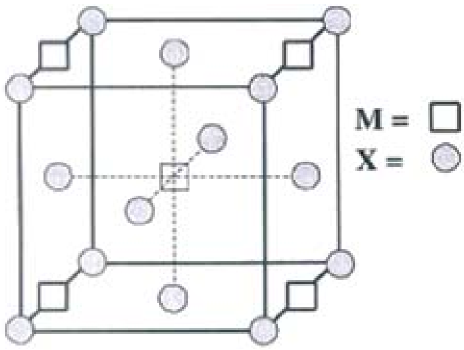
(A) 0 (B) 1 (C) 2 (D) 4

Ans. **[A]**

Sol.



Q.25 A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is -



(A) MX (B) MX_2 (C) M_2X (D) M_5X_{14}

Ans. **[B]**

Sol. $X_{8\times\frac{1}{8}} + 6 \times \frac{1}{2} = X_4$

$M_{4\times\frac{1}{4}} + 1 = M_2$

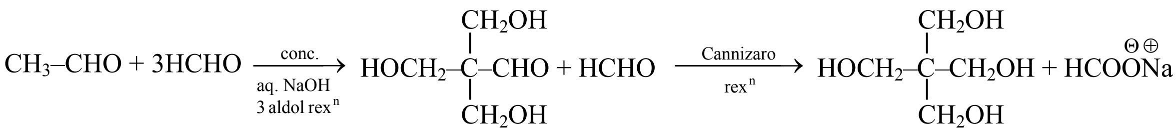
Simplest ration = X_2M

Q.26 The number of aldol reaction (s) that occurs in the given transformation is -

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **[C]**

Sol.



Q.27 The colour of light absorbed by an aqueous solution of $CuSO_4$ is -

(A) orange-red (B) blue-green (C) yellow (D) violet

Ans. **[A]**

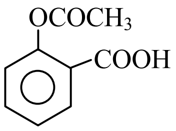
Sol. Factual.

Q.28 The carboxyl functional group (–COOH) is present in -

(A) picric acid (B) barbituric acid (C) ascorbic acid (D) aspirin

Ans. **[D]**

Sol.



Aspirin

Q.29 The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\frac{h^2}{4\pi^2 ma_0^2}$ (B) $\frac{h^2}{16\pi^2 ma_0^2}$ (C) $\frac{h^2}{32\pi^2 ma_0^2}$ (D) $\frac{h^2}{64\pi^2 ma_0^2}$

Ans. **[C]**

Sol. $mvr = \frac{nh}{2\pi}$

$mv = \frac{nh}{2\pi r}$

$m^2v^2 = \frac{n^2h^2}{4\pi^2 r^2}$

$\frac{1}{2} m^2v^2 = \frac{n^2h^2}{8\pi^2 r^2}$

$\frac{1}{2}\frac{m^2v^2}{m} = K.E = \frac{n^2h^2}{8\pi^2 m \times \left(\frac{n^2}{z}\right)^2 a_0^2}$

$= \frac{h^2}{8\pi^2 m \times n^2\, a_0^2}$ $\left\{\begin{matrix} n = 2 \\ z = 1 \end{matrix}\right\}$

$= \frac{h^2}{32\pi^2\, ma_0^2}$

Q.30 Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen ?

(A) HNO_3, NO, NH_4Cl, N_2 (B) HNO_3, NO, N_2, NH_4Cl

(C) HNO_3, NH_4Cl, NO, N_2 (D) NO, HNO_3, NH_4Cl, N_2

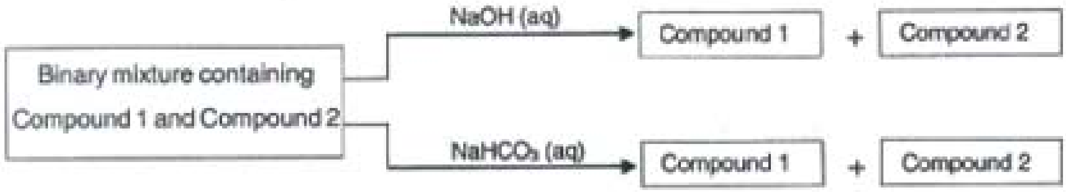
Ans. **[B]**

Sol. $H\overset{+5}{N}O_3$ $\overset{+2}{N}O$ $\overset{0}{N_2}$ $\overset{-3}{N}H_4Cl$

SECTION – II : MULTIPLE CORRECT ANSWER(S) TYPE

This section contains 5 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE are correct.**

Q.31 Identify the binary mixture(s) that can be separated into individual compounds, by differential extraction, as shown in the given scheme.



(A) C_6H_5OH and C_6H_5COOH (B) C_6H_5COOH and $C_6H_5CH_2OH$
(C) $C_6H_5CH_2OH$ and C_6H_5OH (D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

Ans. **[B,D]**

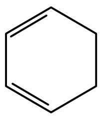
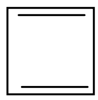
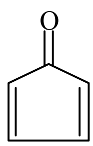
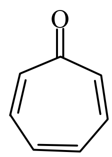
Sol. Factual

Q.32 Choose the correct reason(s) for the stability of the **lyophobic** colloidal particles.
(A) Preferential adsorption of ions of on their surface from the solution
(B) Preferential adsorption of solvent on their surface from the solution
(C) Attraction between different particles having opposite charges on their surface
(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

Ans. **[A,D]**

Sol. Factual.

Q.33 Which of the following molecules, in pure form, is (are) **unstable** at room temperature ?

(A)  (B)  (C)  (D) 

Ans. **[B,C]**

Sol. Antiaromaticity

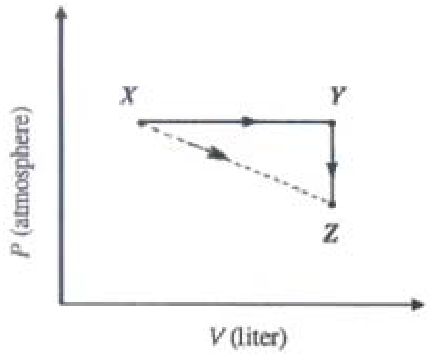
Q.34 Which of the following hydrogen halides react(s) with $AgNO_3(aq)$ to give a precipitate that dissolves in $Na_2S_2O_3(aq)$?
(A) HCl (B) HF (C) HBr (D) HI

Ans. **[A,C,D]**

Sol. $AgBr + 2Na_2S_2O_3 \longrightarrow Na_3[Ag(S_2O_3)_2] + NaBr$
Sodium argentothio sulphate
colourless soluble

Similar reaction observed in AgCl, AgI

Q.35 For an ideal gas, consider only *P*-*V* work in going from an initial state *X* to the final state *Z*. The final state *Z* can be reached by either of the two paths shown in the figure. Which of the following choices(s) is (are) correct ? [take ΔS as change in entropy and *w* as work done]



(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$ (B) $W_{x\to z} = W_{x\to y} + W_{y\to z}$

(C) $W_{x\to y\to z} = W_{x\to y}$ (D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

Ans. **[A,C]**

Sol. $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$

State

$W_{x\to y\to z} = W_{x\to y} + W_{y\to z} \to 0$

SECTION – III : INTEGER ANSWER TYPE

This section contains 5 **questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive)

Q.36 The substituents $\mathbf{R_1}$ and $\mathbf{R_2}$ for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0 ?

$$\overset{\oplus}{H_3N}\text{–}\underset{\substack{|\\H}}{CH}\text{–CO–NH–}\underset{\substack{|\\R_1}}{CH}\text{–CO–NH–}\underset{\substack{|\\R_2}}{CH}\text{–CO–NH–}\underset{\substack{|\\H}}{CH}\text{–}\overset{\ominus}{COO}$$

Peptide	$\mathbf{R_1}$	$\mathbf{R_2}$
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2 COOH	CH_2CONH_2
VIII	CH_2 OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4$ NH_2	CH_3

Ans. **[4]**

Sol. IV, VI, VIII, IX

Four amides are positively charged at pH = 7.0

Q.37 The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element **X** as shown below. To which group, element **X** belongs in the periodic table ?

$$^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + \mathbf{X}$$

Ans. **[8]**

Sol. Z = 29 + 1 – 2 – 2 = 26

Fe $\rightarrow$ 8th groups.

Q.38 When the following aldohexose exists in its **D**-configuration, the total number of stereoisomers in its pyranose form is -

$$\begin{array}{l} CHO \\ | \\ CH_2 \\ | \\ CHOH \\ | \\ CHOH \\ | \\ CHOH \\ | \\ CH_2OH \end{array}$$

Ans. **[8]**

Sol. 8 stereoisomers possible for given compound in pyranose form of D-configuration.

Q.39 29.2% (w/w) HCl stock solution has a density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5 g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 M HCl is.

Ans. **[8]**

Sol. $$\frac{x \times d \times 10}{\text{mol. wt}} \times V = 200 \times 0.4$$

$$\frac{29.2 \times 1.25 \times 10 \times V}{36.5} = 200 \times 0.4$$

V = 8 ml.

Q.40 An organic compound undergoes first-order decomposition. The time taken for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\frac{[t_{1/8}]}{[t_{1/10}]} \times 10$? (take $\log_{10}2 = 0.3$)

Ans. **[9]**

Sol. $$\frac{t_{1/8}}{t_{1/10}} \times 10 = \frac{\frac{2.303}{K}\log\left(\frac{a}{a/8}\right)}{\frac{2.303}{K}\log\left(\frac{a}{a/10}\right)} \times 10 = 9$$

Part – III : (MATHEMATICS)

SECTION – I (Single Correct Answer Type)

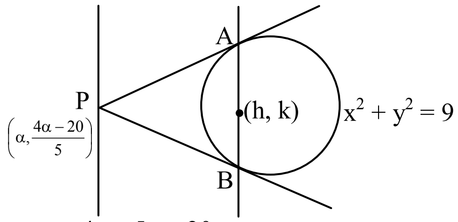
This section contains 10 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Q.41 The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$
(B) $20(x^2 + y^2) + 36x - 45y = 0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$
(D) $36(x^2 + y^2) + 20x - 45y = 0$

Ans. **[A]**

Sol.



4x – 5y = 20

Equation of chord AB is T = 0

$$\alpha x + \left(\frac{4\alpha - 20}{5}\right) y = 9 \qquad \text{...(i)}$$

$$\& \ hx + ky - 9 = h^2 + k^2 - 9 \qquad \text{...(ii)}$$

$\because$ Equation (i) & (ii) both represent the same line

$$\text{So } \frac{\alpha}{h} = \frac{\frac{4\alpha - 20}{5}}{k} = \frac{9}{h^2 + k^2}$$

$$\alpha = \frac{9h}{h^2 + k^2} = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$$36h = 45k + 20(h^2 + k^2)$$

$$20(x^2 + y^2) - 36x + 45y = 0$$

Q.42 The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150 (C) 210 (D) 243

Ans. **[B]**

Sol.

G_1	G_2	G_3
1	1	3
1	2	2

$$\left(\frac{5!}{1!\,1!\,3!\,2!} + \frac{5!}{1!\,2!\,2!\,2!}\right) 3!$$

$= 150$

Q.43 Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in IR$,

then f is

(A) differentiable both at x = 0 and at x = 2

(B) differentiable at x = 0 but not differentiable at x = 2

(C) not differentiable at x = 0 but differentiable at x = 2

(D) differentiable neither at x = 0 nor at x = 2

Ans. **[B]**

Sol. $f'(0+h) = \lim_{h\to 0} \frac{h^2\left|\cos\frac{\pi}{h}\right| - 0}{h-0} = 0$

$f'(0-h) = \lim_{h\to 0} \frac{h^2\left|\cos\frac{\pi}{h}\right| - 0}{-h} = 0$

$\because f'(0^+) = f'(0^-) = 0 = \text{finite}$

So f(x) is differentiable at x = 0

$f'(2+h) = \lim_{h\to 0} \frac{(2+h)^2\left|\cos\left(\frac{\pi}{2+h}\right)\right| - 0}{h} = \pi$

$f'(2-h) = \lim_{h\to 0} \frac{(2-h)^2\left|\cos\left(\frac{\pi}{2-h}\right)\right| - 0}{-h} = -\pi$

$\because f'(2^+) \neq f'(2^-)$ but both are finite so f(x) is not differentiable at x = 2 but continuous at x = 2

Q.44 The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one-one and onto.

(B) onto but not one-one.

(C) one-one but not onto.

(D) neither one-one nor onto.

Ans. **[B]**

Sol. Given $f : [0, 3] \to [1, 29]$

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x - 2)(x - 3)$

$f'(x) > 0$ if $x \in (0, 2)$

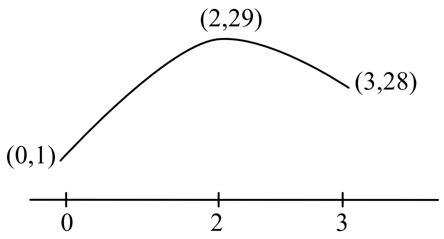
& $f'(x) < 0$ if $x \in (2, 3)$

$\therefore$ Function is many one & continuous

Now f(0) = 1

f(2) = 29

$\therefore$ Range = co-domain



Hence function is onto.

Q.45 If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) a = 1, b = 4 (B) a = 1, b = – 4 (C) a = 2, b = –3 (D) a = 2, b = 3

Ans. **[B]**

Sol. $\lim_{x\to\infty}\left(\frac{x^2+x+1-ax^2-ax-bx-b}{x+1}\right)=4$

$\lim_{x\to\infty}\left(\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}\right)=4$

As limit is finite so 1 – a = 0

$\Rightarrow$ a = 1

Now $\lim_{x\to\infty}\left(\frac{(1-a-b)+\frac{1-b}{x}}{1+\frac{1}{x}}\right)=4$

$\Rightarrow 1-a-b=4$

as a = 1 $\Rightarrow$ b = –4

Q.46 Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a **cannot** take the value

(A) –1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans. **[D]**

Sol. As a is real

So $a = \bar{a}$

$\Rightarrow z^2+z+1=\bar{z}^2+\bar{z}+1$

$\Rightarrow (z-\bar{z})(z+\bar{z}+1)=0$

As z is imaginary

So $z-\bar{z}\neq 0$

$\Rightarrow z+\bar{z}+1=0$

$\Rightarrow z + \bar{z} = -1 \;\; \forall\, z = x + iy$

$x = \frac{-1}{2}$

so $a = (x + iy)^2 + (x + iy) + 1$

$= (x^2 + x + 1 - y^2) + (2x + 1)yi \;\; \forall\, x = -\frac{1}{2}$

$a = \frac{3}{4} - y^2$

so $a < \frac{3}{4} \;\; \forall\, y^2 > 0$

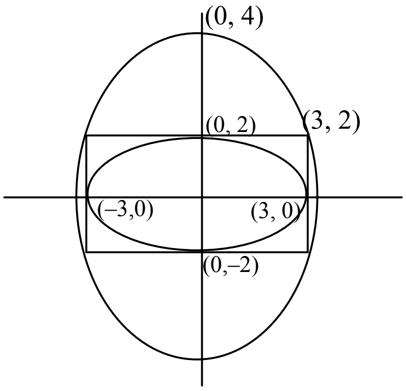
So $a \neq \frac{3}{4}$

Q.47 The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans. **[C]**

Sol.



Let equation of ellipse E_2 is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

it passes through (0, 4)

so $b^2 = 16$

and also passes through (3, 2)

So $\frac{9}{a^2} + \frac{4}{b^2} = 1 \quad \Rightarrow \frac{9}{a^2} + \frac{1}{4} = 1 \quad \Rightarrow a^2 = 12$

$\Rightarrow$ as $a < b$

so $12 = 16 (1 - e^2)$

$\Rightarrow e^2 = \frac{1}{4} \qquad \Rightarrow e = \frac{1}{2}$

Q.48 Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \le i, j \le 3$, If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

Ans. **[D]**

Sol. Let $P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

So $Q = \begin{bmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{bmatrix}$

Now det (Q) = $2^2 2^3 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33} \end{vmatrix}$

$= 2^9 \times 2 \times 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= 2^{12}$ det (P)

$= 2^{12} \times 2 = 2^{13}$

49. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}}\, dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

Ans. **[C]**

Sol. $I = \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$

sex x + tan x = t

$(\sec x \tan x + \sec^2 x)\, dx = dt$

$\sec x\,dx = \frac{dt}{t}$

also, $\sec x - \tan x = \frac{1}{t}$

$\sec x = \frac{1}{2}\left(t + \frac{1}{t}\right)$

So, $I = \frac{1}{2}\int \frac{\left(t + \frac{1}{t}\right)dt}{t^{11/2}}$

$= \frac{1}{2}\int \left(\frac{1}{t^{9/2}} + \frac{1}{t^{13/2}}\right) dt$

$= \frac{1}{2}\left[-\frac{2}{7}\frac{1}{t^{7/2}} - \frac{2}{11}\frac{1}{t^{11/2}}\right] + K$

$= -\frac{1}{t^{11/2}}\left[\frac{t^2}{7} + \frac{1}{11}\right] + K$

$= -\frac{1}{(\sec x + \tan x)^{11/2}}\left[\frac{1}{7}(\sec x + \tan x)^2 + \frac{1}{11}\right] + K$

50 The point P is the intersection of the straight line joining the points $Q(2,3,5)$ and $R(1,-1,4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2,1,4)$ to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Ans. **[A]**

Sol. Equation of line

$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1} = \lambda$

General points $\{\lambda + 2, 4\lambda + 3, \lambda + 5\}$

Intersection point with plane

$5(\lambda + 2) - 4(4\lambda + 3) - (\lambda + 5) = 1$

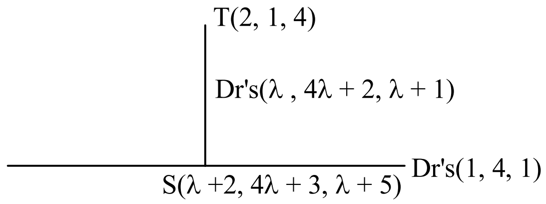
$5\lambda + 10 - 16\lambda - 12 - \lambda - 5 = 1$

$-12\lambda - 8 = 0$

$\lambda = -\frac{8}{12} = -\frac{2}{3}$

Point $\left[\frac{-2}{3} + 2, -\frac{8}{3} + 3, \frac{-2}{3} + 5\right]$

$P\left[\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right]$



Now

$\lambda + 4\,(4\lambda + 2) + (\lambda + 1) = 0$

$\lambda + 16\lambda + 8 + \lambda + 1 = 0$

$18\lambda = -9$

$\lambda = -\frac{1}{2}$

Points $\left(\frac{-1}{2} + 2,\ -2 + 3,\ -\frac{1}{2} + 5\right)$

$\left(\frac{3}{2},\ 1,\ \frac{9}{2}\right)$

Distance at PS = $\sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \left(\frac{1}{3} - 1\right)^2 + \left(\frac{13}{3} - \frac{9}{2}\right)^2}$

PS = $\sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}} = \sqrt{\frac{1+16+1}{36}} = \sqrt{\frac{18}{36}} = \frac{1}{\sqrt{2}}$

SECTION – II (Multiple Correct Answer(s) Type)

This section contains 5 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE** are correct.

51. Let $\theta, \varphi \in [0, 2\pi]$ be such that

$$2\cos\theta\,(1 - \sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1,$$

$\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$.

Then φ **cannot** satisfy

(A) $0 < \varphi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \varphi < 2\pi$

Ans. **[A, C, D]**

Sol. $\tan(2\pi - \theta) > 0$

$\Rightarrow 0 < 2\pi - \theta < \frac{\pi}{2}$ or $\pi < 2\pi - \theta < \frac{3\pi}{2}$

$\Rightarrow \frac{3\pi}{2} < \theta < 2\pi$ or $\frac{\pi}{2} < \theta < \pi$ ...... (1)

Also $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$

$\frac{3\pi}{2} < \theta < \frac{5\pi}{3}$ .....(2)

from (1) & (2)

$\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$ .....(3)

Now, $2\cos\theta\,(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$

$\Rightarrow \cos\theta + \frac{1}{2} = \sin(\theta + \varphi)$ ....(4)

Now, $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$ from (3)

so $\cos\theta \in \left(0, \frac{1}{2}\right)$

$\sin(\theta + \varphi) \in \left(\frac{1}{2}, 1\right)$

Now, check option

(A) if $0 < \varphi < \frac{\pi}{2}$

then $\theta + \varphi \in \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$ & $\sin(\theta + \varphi) \notin \left(\frac{1}{2}, 1\right)$

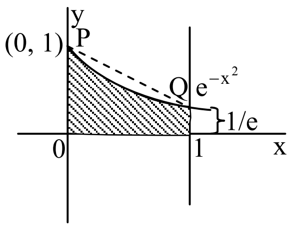
Similarly check option B, C, D.

52. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

(A) $S \geq \frac{1}{e}$ (B) $S \geq 1 - \frac{1}{e}$

(C) $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Ans. **[A, B, D]**



Sol.

$$\int_0^1 e^{-x^2} dx > \int_0^1 e^{-x} dx$$

$$= 1 - \frac{1}{e} \text{ (A), (B)}$$

Area above x-axis by PQ line $y = 1 + x\left(\frac{1}{e} - 1\right)$

$$S \leq \int_0^1 y\, dx = \frac{1+e}{2e} < \text{(D) also (B) > (C)}$$

Hence (C) not possible.

Hence A, B, D

53. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$ (B) P [Exactly two engines of the ship are functioning $\mid X$] $= \frac{7}{8}$

(C) $P\left[X \mid X_2\right] = \frac{5}{16}$ (D) $P\left[X \mid X_1\right] = \frac{7}{16}$

Ans. [B,D]

Sol. $P(X) = P(\overline{X}_1 \cap X_2 \cap X_3) + P(X_1 \cap \overline{X}_2 \cap X_3) + P(X_1 \cap X_2 \cap \overline{X}_3) + P(X_1 \cap X_2 \cap X_3)$

$$P(X) = \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4}$$

$$= \frac{1}{4}$$

$$P\left(\frac{\overline{X}_1}{X}\right) = \frac{P(\overline{X}_1 \cap X)}{P(X)} = \frac{P(\overline{X}_1 \cap X_2 \cap X_3)}{P(X)}$$

$$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

$$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)}$$

$$= \frac{P(X_1).P(X_2).P(X_3) + P(X_1).P(X_2).P(\overline{X}_3) + P(\overline{X}_1).P(\overline{X}_2).P(X_3)}{P(X_2)}$$

$$= P(X_1).P(X_3) + P(X_1).P(\overline{X}_3) + P(\overline{X}_1).P(X_3)$$

$$= \frac{1}{2}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4} + \frac{1}{2}.\frac{1}{4} = \frac{5}{8}$$

$$P\left(\frac{X}{X_1}\right) = P(X_2).P(X_3) = P(\overline{X}_2)P(X_3) + P(X_2).P(\overline{X}_3)$$

$$= \frac{1}{4}.\frac{1}{4} + \frac{3}{4}.\frac{1}{4} + \frac{1}{4}.\frac{3}{4}$$

$$= \frac{7}{16}$$

$$P\left(\frac{\text{Exactly two engine are working}}{X}\right) = \frac{P\left(\frac{\text{Exactly two engine are working}}{X} \cap X\right)}{P(X)}$$

$$= \frac{P(X_1)P(X_2)P(\overline{X}_3) + P(\overline{X}_1)P(X_2)P(X_3) + P(X_1)P(\overline{X}_2)P(X_3)}{P(X)}$$

$$= \frac{\frac{7}{32}}{\frac{1}{4}} = \frac{7}{8}$$

54 Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3},-2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3},2\sqrt{2}\right)$

Ans. [A, B]

Sol. Equation of tangent is

$2x - y + c = 0$

$y = 2x + c$

slope $m = 2$

$\therefore\ a^2 = 9, b^2 = 4$

$\therefore\ c^2 = a^2m^2 - b^2 = 9 \times 4 - 4$

$c = \pm\ 4\sqrt{2}$

$\therefore$ point of contact is $\left(\pm\ \frac{a^2m}{c}, \pm\frac{b^2}{c}\right)$

$\left(\pm\ \frac{9}{2\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$

55. If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0)=0$, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Ans. **[A,D]**

Sol. $\frac{dy}{dx} - y\tan x = 2x\sec x$

$P = \tan x$

I.F. $= e^{-\int \tan x dx} = \cos x$

$y.\cos x = \int 2x.dx$

$y\cos x = x^2 + C$

$\because\ y(0) = 0 \Rightarrow C = 0$

$y = x^2 \sec x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ $\qquad$ $y\left(\frac{\pi}{3}\right) = \frac{2\pi^2}{9}$

$y'(\pi) = 2x \sec x + x^2 \sec x \tan x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{\sqrt{2}} + \frac{\pi^2}{16}\sqrt{2} = \frac{\pi}{\sqrt{2}} + \frac{\pi^2}{8\sqrt{2}} = \frac{9\pi^2}{8\sqrt{2}}$

$y'\left(\frac{\pi}{3}\right) = \frac{2\pi}{3} \times 2 + \frac{\pi^2}{9} 2.\sqrt{3}$

SECTION – III (Integer Answer Type)

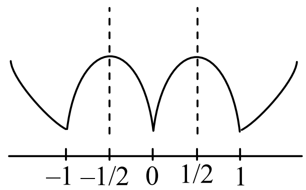
This section contains 5 **questions**. The answer to each question is a **single digit integer,** ranging from 0 to 9 (*both inclusive).*

56. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Ans. **[5]**

Sol. $f(x) = |x| + |x - 1||x + 1|$

$x \geq 1$	$f(x) = x^2 + x - 1$	$f'(x) = 2x + 1$	+ve
$0 \leq x < 1$	$f(x) = 1 - x^2 + x$	$f'(x) = 1 - 2x$	$x > \frac{1}{2}$ –ve
$-1 < x < 0$	$f(x) = 1 - x^2 - x$	$f'(x) = -2x - 1$	$x > -\frac{1}{2}$ –ve ; $x < -\frac{1}{2}$ +ve
$x \leq -1$	$f(x) = x^2 - x - 1$	$f'(x) = 2x - 1$	–ve



57 The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right)$ is

Ans. **[4]**

Sol. Let $x = \sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}$

$x^2 = 4 - \frac{1}{3\sqrt{2}} x$

$3\sqrt{2}\, x^2 + x - 12\sqrt{2} = 0$

$$x = \frac{-1+\sqrt{1+4.3\sqrt{2}.12\sqrt{2}}}{6\sqrt{2}}$$

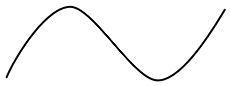
$$x = \frac{-1+17}{6\sqrt{2}} = \frac{8}{3\sqrt{2}}$$

$$6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\frac{3}{2}\right)^{-2} = 6 - 2 = 4$$

58. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is

Ans. [9]

Sol. $P'(1) = 0,\ P'(3) = 0$



$P'(x) = K(x-1)(x-3)$

$= K(x^2 - 4x + 3)$ $\qquad$ $P'(0) = 3K$

$P(x) = \frac{K}{3}x^3 - 2Kx^2 + 3Kx + \lambda$

$\frac{K}{3} - 2K + 3K + \lambda = 6,$ $\qquad$ $9K - 18K + 9K + \lambda = 2$

$\frac{4}{3}K + \lambda = 6,$ $\qquad$ $\frac{4}{3}K = 4$

$K = 3$

$P'(0) = 9$

59. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Ans. [3]

Sol. $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$

$a.b + b.c + c.a = -3/2$ $\qquad$ ....(1)

$\because$ $|\vec{a}+\vec{b}+\vec{c}|^2 \geq 0$ $\qquad$ ....(2)

$a.b + b.c + c.a \geq \frac{-3}{2}$ $\qquad$ ....(3)

$\because$ from (1) & (3)

so $|\vec{a}+\vec{b}+\vec{c}| = 0$

$\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{a} = -\vec{b}-\vec{c}$

on squaring

$1 = 2 + 2\cos B$

$\cos B = -\frac{1}{2} \quad \forall\, B = \vec{b} \wedge \vec{c}$.

Let $T = |2\vec{a} + 5\vec{b} + 5\vec{c}|$

$= |3\vec{b} + 3\vec{c}|$

$= 3|\vec{b}+\vec{c}|$

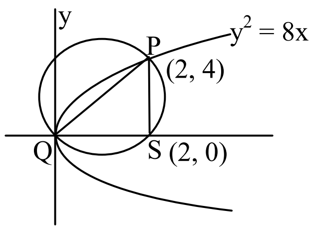
$= 3\sqrt{2+2\cos B}$

$= 3$

60. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of the triangle PQS is

Ans. [4]

Sol. $(x-1)^2 + (y-2)^2 = (\sqrt{5})^2$



$x^2 + 8x - 2x - 4.2\sqrt{2x} = 0$

$x^2 + 6x - 8\sqrt{2x} = 0$

$x^{3/2} + 6x^{1/2} - 8\sqrt{2} = 0$

$x^{1/2} = t$

$t^3 + 6t - 8\sqrt{2} = 0$

$(t-\sqrt{2})(t^2 - \sqrt{2}\,t + 4) = 0$

$t = \sqrt{2} \qquad x = 2$

$y = 4$

P(2, 4) Q(0, 0) S(2, 0)

Area $\Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$