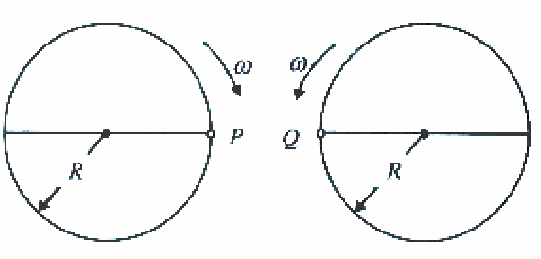
CODE : 8 08 / 04 / 2012

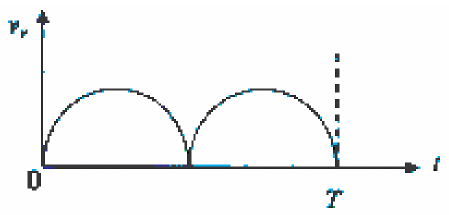
Part – I : (PHYSICS)

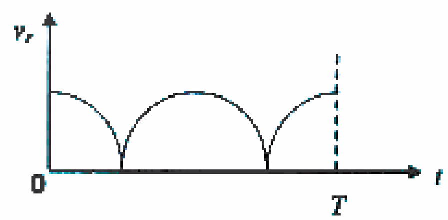
SECTION – I (Single Correct Answer Type)

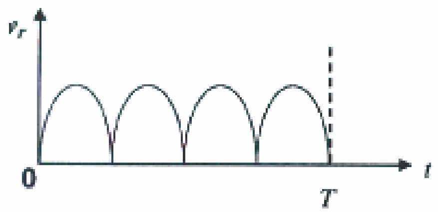
This section contains 8 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

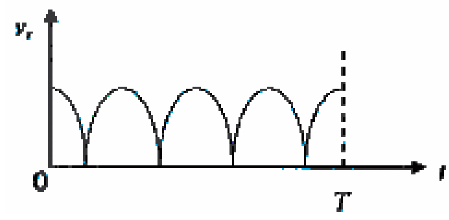
Q.1 Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by -



(A) 

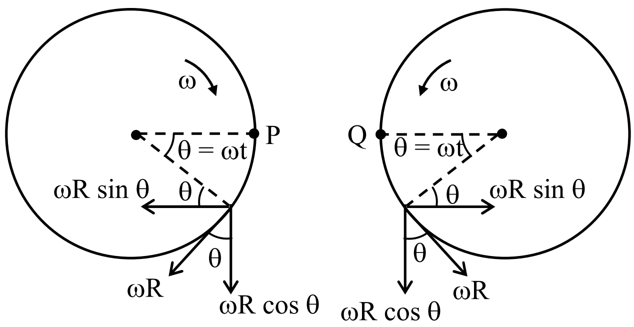
(B) 

(C) 

(D) 

Ans. **[A]**

Sol.

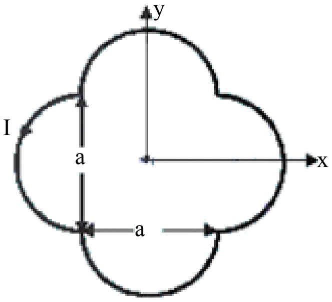


So, $v_r = 2\omega R \sin(\omega t)$

At $t = T/2$, $v_r = 0$

So two half cycles will take place.

Q.2 A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is -



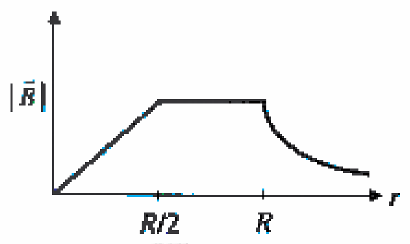
(A) $a^2I\ \hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2I\ \hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2I\ \hat{k}$ (D) $(2\pi + 1)\ a^2I\ \hat{k}$

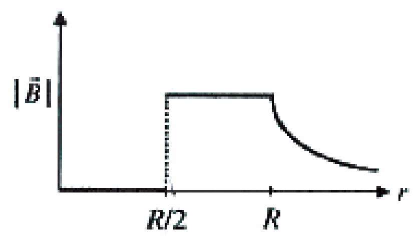
Ans. **[B]**

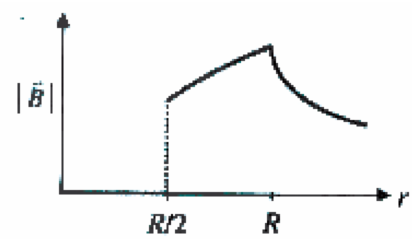
Sol. $M = I \times \text{Area of loop } \hat{k}$

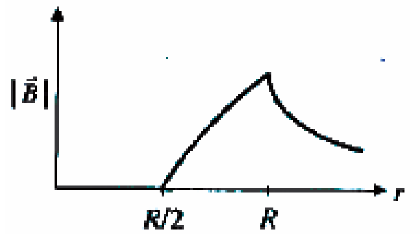
$$= I \times \left[a^2 + \frac{\pi a^2}{4\times 2}\times 4\right]\hat{k} = I\times a^2\left[\frac{\pi}{2}+1\right]\hat{k}$$

Q.3 An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by -

(A)


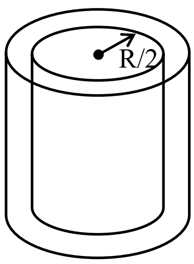
(B)


(C)


(D)


Ans. **[D]**

Sol.



$r < \frac{R}{2}$; $B = 0$

B at $r = \frac{R}{2}$

$$B = \frac{\mu_0 JR}{2\times 2} - \frac{\mu_0 JR}{2\times 2} = 0$$

B at $r > \frac{R}{2}$

$$B = \frac{\mu_0 JR}{2} - \frac{\mu_0 J \times \pi}{2\pi r} \times \frac{R^2}{4}$$

$$B = \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right]$$

if we put $r = \frac{R}{2}$

$B = 0$

$\therefore$ B is continuous at r = R /2

Q.4 A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is -

(A) more than half-filled if ρ_c is less than 0.5

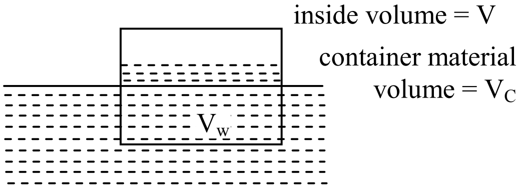
(B) more than half-filled if ρ_c is less than 1

(C) half-filled if ρ_c is more than 0.5

(D) less than half-filled if ρ_c is less than 0.5

Ans. **[A]**

Sol.



$m_C g + m_w g = F_B$

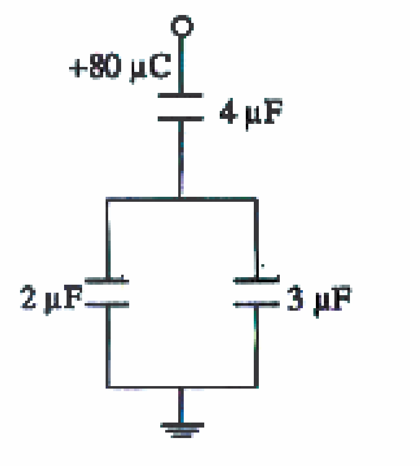
$$\rho_C V_C g + 1\ V_w g = 1\left[\frac{V}{2} + \frac{V_C}{2}\right] g$$

$$V_w = \frac{V}{2} + V_C\left[\frac{1}{2} - \rho_C\right]$$

if $\rho_C < \frac{1}{2}$; $V_w > \frac{V}{2}$

correct ans. "A"

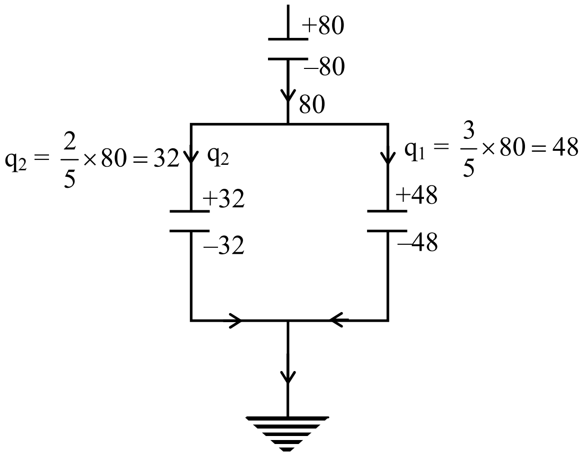
Q.5 In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is



(A) +32μC (B) +40μC (C) +48μC (D) +80μC

Ans. **[C]**

Sol.



2μF & 3μF are in parallel combination

Q.6 Two moles of ideal helium gas are in a rubber balloon at 30ºC. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35ºC. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

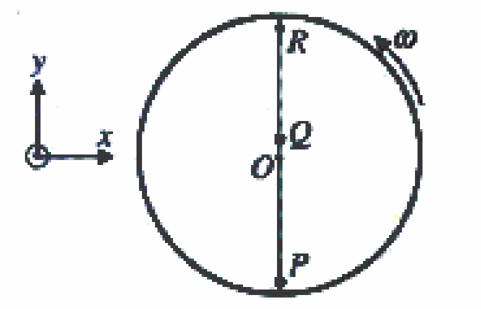
(A) 62 J (B) 104 J (C) 124 J (D) 208 J

Ans. **[D]**

Sol. When balloon is expanding slowly. It is expanding against atmospheric pressure at constant pressure process.

$$\Delta Q = \Delta W + \Delta U$$
$$= nR\Delta T + nC_V\Delta T$$
$$\Delta Q = 2\left[R + \frac{3R}{2}\right] \times \Delta T = 5 \times 3.14 \times 5 = 208 \text{ J}$$

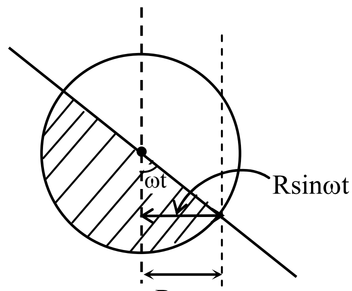
Q.7 Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation (ii) their range is less than half the disc radius and (iii) ω remains constant throughout. Then



(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

Ans. **[C or D]**

Sol.



According to problem particle is to land on disc.
If one consider a time 't' then x component of disc is Rωt
Rsinωt < Rωt

This particle 'P' land on unshaded region. For "Q" x-component is very small and *y*-component equal to P it will also land in unshaded region.

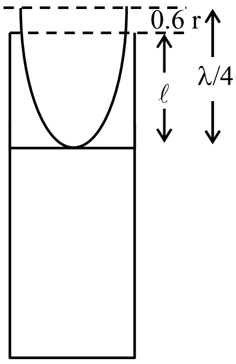
Now repeat same thing when right part is shaded then correct answer is "C" or "D"

Q.8 A student performing the experiment of Resonance Column. The diameter of the column tube is 4cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38ºC in which the speed of sound is 336 m/s The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

Ans. **[B]**

Sol.



$f\lambda = v$

$\lambda = \frac{336}{512}$ m ; $\frac{\lambda}{4} = 0.6 \times r + \ell$

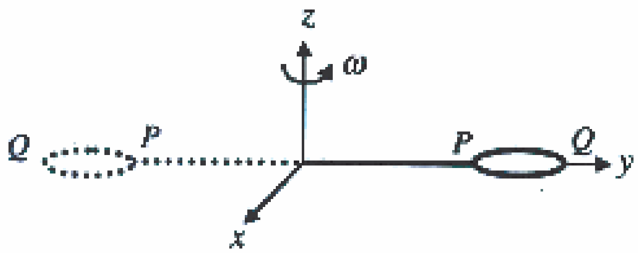
$\frac{336}{512 \times 4} \times 100 = 1.2 + \ell$; $\ell = 15.2$ cm

SECTION – II (Paragraph Type)

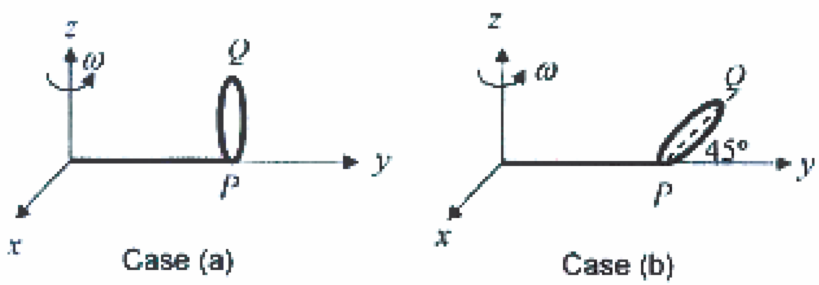
This section contains 6 **multiple choice questions** relating to three paragraphs with **two questions on each paragraph**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Questions 9 and 10

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this chase.



Now consider two similar systems as shown in the figure : Case (a) the disc with its face vertical and parallel to x-y plane; Case (b) the disc with its face making an angle of 45º with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



Q.9 Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct ?

(A) It is vertical for both the cases (a) and (b)

(B) It is vertical for case (a); and is at 45º to the x-z plane and lies in the plane of the disc for case (b)

(C) It is horizontal for case (a); and is at 45º to the x-z plane and is normal to the plane of the disc for case (b)

(D) It is vertical for case (a); and is at 45º to the x-z plane and is normal to the plane of the disc for case (b)

Ans. **[A]**

Sol. In both the cases, the instantaneous axis will be along z-axis i.e. along vertical direction

Q.10 Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct ?

(A) It is $\sqrt{2}\,\omega$ for both the cases

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)

(C) It is ω for case (a); and $\sqrt{2}\,\omega$ for case (b)

(D) It is ω for both the cases

Ans. **[D]**

Sol. w.r.t. centre of mass only pure rotation of disc will be seen. So in both the cases, angular speed about instantaneous axis will be ''ω''

Paragraph for Questions 11 and 12

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three body decay process, i.e. $n \to p + e^- + \bar{v}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti neutrino $(\bar{v}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

Q.11 If the anti-neutrino had a mass of $3eV/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron ?

(A) $0 \le K \le 0.8 \times 10^6$ eV

(B) $3.0\text{ eV} \le K \le 0.8 \times 10^6$ eV

(C) $3.0\text{ eV} \le K < 0.8 \times 10^6$ eV

(D) $0 \le K \le 0.8 \times 10^6$ eV

Ans. **[D]**

Sol. Total energy remain conserved. Energy is shared by antineutrino, proton and electron.

KE of e^- is maximum when antineutrino does not share any KE.

Now total energy is shared with proton & electron.

$\therefore$ $K < 0.8 \times 10^6$ eV

Minimum KE of e^- can be zero when total energy is shared by proton and antineutrino.

$\therefore$ $0 \le K \le 0.8 \times 10^6$ eV

Q.12 What is the maximum energy of the anti-neutrino ?

(A) Zero

(B) Much less than 0.8×10^6 eV

(C) Nearly 0.8×10^6 eV

(D) Much larger than 0.8×10^6 eV.

Ans. **[C]**

Sol. When e^- has zero kinetic energy total energy is shared by antineutrino and proton. This time energy of antineutrino is its maximum possible kinetic energy.

As antineutrino is very light mass in comparison to proton so it will have almost contribution in total energy.

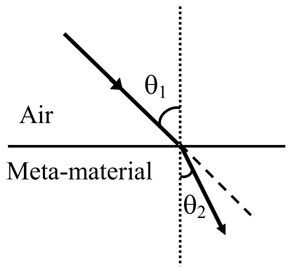
$\therefore$ its energy is almost 0.8×10^6 eV

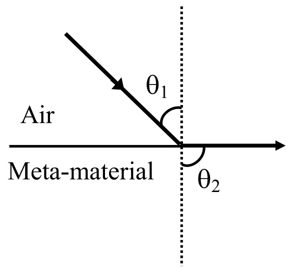
Paragraph for Questions 13 and 14

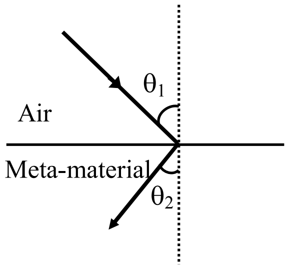
Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

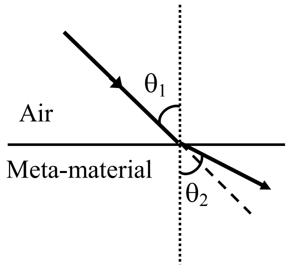
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

Q.13 For light incident from air on a meta-material, the appropriate ray diagram is -

(A)


(B)


(C) 

(D) 

Ans. **[C]**

Sol. As per snell's law,

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

So most probable answer is option ''C''.

Q.14 Choose the correct statement.

(A) The speed of light in the meta material is $v = c|n|$

(B) The speed of light in the meta material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is $v = c$

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air

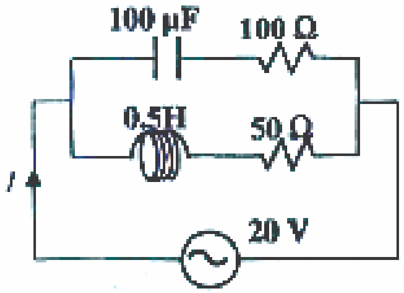
Ans. **[B]**

Sol. The expression $v = \frac{c}{|n|}$ is applicable.

SECTION – III (Multiple Correct Answer(s) Type)

This section contains 6 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE** are correct.

Q.15 In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are)



(A) The current through the circuit, I is 0.3 A

(B) The current through the circuit, I is $0.3\sqrt{2}$ A

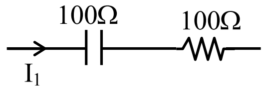
(C) The voltage across 100 Ω resistor = $10\sqrt{2}$ V

(D) The voltage across 50 Ω resistor = 10 V

Ans. **[A, C] or [C]**

Sol. $x_L = \omega L = 10 \times 0.5 = 50\ \Omega$

$$x_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\Omega$$



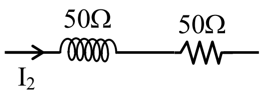
$Z_1 = 100\sqrt{2}$; $I_1 = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}$

$$V_{across}\ 100\ \Omega = \frac{1}{5\sqrt{2}} \times 100 = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 10\sqrt{2}\ \text{Ans. (C)}$$

Phase diff. between I_1 & V $\Rightarrow$ $\cos\phi_1 = \frac{R_1}{Z_1} = \frac{100}{100\sqrt{2}}$

$\phi_1 = \pi/4$

I_1 lead V



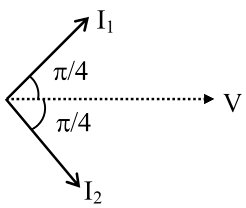
$Z_2 = 50\sqrt{2}$; $I_2 = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}}$

$$V_{rms}\ \text{across}\ 50\Omega = \frac{2}{50\sqrt{2}} \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{Ans.}$$

$\phi_2 = \pi/4$

I_2 lag V by $\pi/4$

$I = I_1 + I_2$



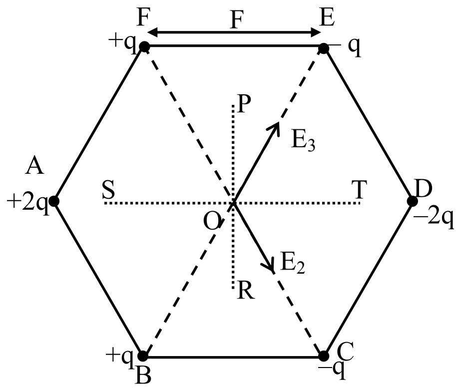
$$I_{Net} = \sqrt{I_1^2 + I_2^2}$$

$$I = \sqrt{\frac{4}{25 \times 2} + \frac{1}{25 \times 2}} = \sqrt{\frac{5}{50}} = \frac{1}{\sqrt{10}}$$

$I = 0.316$

As I is not exactly 0.3 therefore IIT give answer either C or (A,C)

Q.16 Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct ?



(A) The electric field at O is 6K along OD.

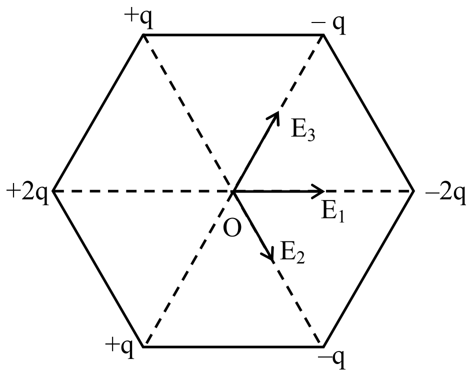
(B) The potential at O is zero

(C) The potential at all points on the line PR is same

(D) The potential at all points on the line ST is same

Ans. **[A, B, C]**

Sol.



$E_2 = E_3 = \frac{2kq}{L^2}$

$E_1 = \frac{4kq}{L^2}$

$E_{all} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$= \frac{6kq}{L^2} = 6 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$ = 6K (Along OD)

V_{at} O = 0

V_{at} line PR is zero because this line is equatorial axis for three dipole.

Q.17 Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R respectively. Then.

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R / V_P = 3$ (D) $V_P / V_Q = \frac{1}{2}$

Ans. **[B, D]**

Sol. Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$

Surface area of P, $4\pi R_P^2 = A$

Surface area of Q, $4\pi R_Q^2 = 4A$

$\Rightarrow \frac{R_P}{R_Q} = \frac{1}{2}$

$\Rightarrow R_Q = 2R_P$

If $M_P = M = \rho\frac{4}{3}\pi R^3$ (Let R be the radius of P)

So, $M_Q = \rho\frac{4}{3}\pi(2R)^3 = 8M$

So, $M_R = M_P + M_Q = 9M$

For planet R, $9M = \rho\frac{4}{3}\pi R_r^3$

So, $R_r = (9)^{1/3}R$

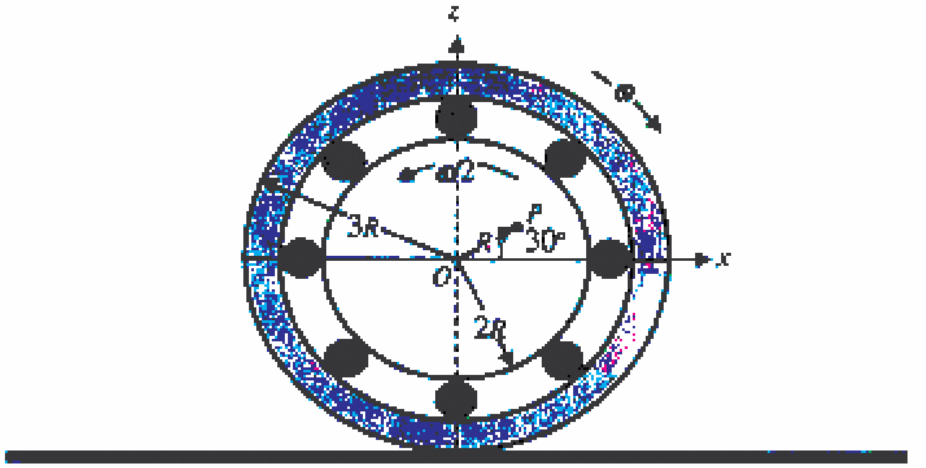
Escape velocity from P, $V_P = \sqrt{\frac{2GM}{R}}$

$V_Q = \sqrt{\frac{2G(8M)}{2R}} = 2\sqrt{\frac{2GM}{R}}$

$V_R = \sqrt{\frac{2G(9M)}{(9)^{1/3}R}} = (9)^{1/3}\sqrt{\frac{2GM}{R}}$

So, $V_R > V_Q > V_P$ & $\frac{V_P}{V_Q} = \frac{1}{2}$

Q.18 The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30º with the horizontal. Then with respect to the horizontal surface,



(A) the point O has a linear velocity $3R\omega\,\hat{i}$

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

Ans. **[A, B]**

Sol. $V_P = 3R\omega\hat{i} + \frac{\omega}{2}(-\hat{j}) \times (R\cos 30°\hat{i} + R\sin 30°\hat{k})$

$= 3R\omega\hat{i} + \sqrt{3}\frac{\omega}{4}R\hat{k} - \frac{\omega}{4}R\hat{i} = \frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

Q.19 Two solid cylinders P and Q of same mass and same radius start rolling down a flixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is (are) correct ?

(A) Both cylinders P and Q reach the ground at the same time.

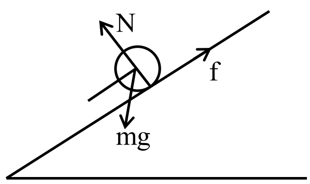
(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinders reach the ground with same translational kinetic energy

(D) Cylinder Q reaches the ground with larger angular speed

Ans. **[D]**

Sol.



Translation motion :

$mg\sin\theta - f = ma_{cm}$ ....(i)

Rotational motion

$fR = I_{cm}\,\alpha$ ....(ii)

Rolling without slipping

$\alpha R = a_{cm}$ ....(iii)

From (ii) & (iii)

$f = \frac{I_{cm}a_{cm}}{R^2}$

Put this in (i)

$mg\sin\theta - \frac{I_{cm}a_{cm}}{R^2} = ma_{cm}$

$a_{cm} = \frac{mg\sin\theta}{\left(\frac{I_{cm}}{R^2} + m\right)}$

As $I_P > I_Q$

So $a_P < a_Q$
So, $v_{cm\,(Q)} > v_{cm\,(P)}$
Hence $\omega_Q > \omega_P$

Q.20 A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are)

(A) The emf induced in the loop is zero if the current is constant.

(B) The emf induced in the loop is finite if the current is constant

(C) The emf induced in the loop is zero if the current decreases at a steady rate

(D) The emf induced in the loop is finite if the current decreases at a steady rate

Ans. **[A, C]**
Sol. Total flux associate with loop = 0
$\therefore$ emf = 0 in any case

Part – II : (Chemistry)

SECTION – I (Single Correct Answer Type)

This section contains 8 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Q.21 The major product **H** of the given reaction sequence is -

$$CH_3-CH_2-CO-CH_3 \xrightarrow{\overset{\ominus}{C}N} G \xrightarrow[\text{Heat}]{95\%H_2SO_4} H$$

(A) $CH_3-CH=\underset{\underset{\displaystyle CH_3}{|}}{C}-COOH$ (B) $CH_3-CH=\underset{\underset{\displaystyle CH_3}{|}}{C}-CN$

(C) $CH_3-CH_2-\overset{\overset{\displaystyle OH}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-COOH$ (D) $CH_3-CH=\underset{\underset{\displaystyle CH_3}{|}}{C}-CO-NH_2$

Ans. **[A]**

Sol. $CH_3–CH_2–CO–CH_3 \xrightarrow{\overset{\ominus}{C}N} CH_3–CH_2–\overset{\overset{\displaystyle OH}{|}}{\underset{\underset{\displaystyle CN}{|}}{C}}–CH_3 \xrightarrow[H_2SO_4,\,\Delta]{95\%} CH_3–CH=\underset{\underset{\displaystyle H_3C}{|}}{C}–\underset{\underset{\displaystyle O}{\|}}{C}–OH$

Q.22 $NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behaviour (paramagnetic / diamagnetic). The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively

(A) tetrahedral and tetrahedral (B) square planar and square planar

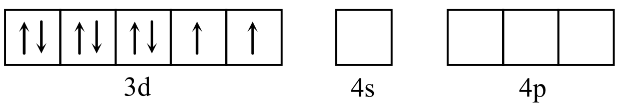
(C) tetrahedral and square planar (D) square planar and tetrahedral

Ans. **[C]**

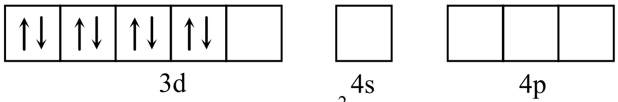
Sol. $\overset{+2}{Ni}\overset{-2}{Cl_2}\{P(C_2H_5)_2^{0}(C_6H_5)\}_2$

$\downarrow$ (Cl₂) negative ligand having –1 charge; $\downarrow$ neutral ligand

Ni ———————— $3d^8 4s^2$
Ni^{+2} ———————— $3d^8 4s^0$



In paramagnetic state there will be no pairing therefore hybridization will be sp^3 and geometry will be tetrahedral.
In diamagnetic state pairing of electron will take place.



Since d-orbital is vacant so, hybridization will be dsp^2 geometry is square planar.

Q.23 In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agent used are

(A) O_2 and CO respectively. (B) O_2 and Zn dust respectively.

(C) HNO_3 and Zn dust respectively. (D) HNO_3 and CO respectively.

Ans. **[B]**

Sol. $4Ag + 8NaCN + 2H_2O + O_2 \rightarrow 4\,NaAg(CN)_2 + 4NaOH$

$2NaAg(CN)_2 + Zn \rightarrow Na_2Zn(CN)_4 + 2Ag$

Q.24 The reaction of white phosphorus with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type ; the oxidation states of phosphorus phosphine and the other product are respectively

(A) redox reaction ; –3 and –5 (B) redox reaction ; +3 and +5

(C) disproportionation reaction ; –3 and +5 (D) disproportionation reaction ; –3 and +3

Ans. **[C]**

Sol. (i) $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2\overset{+1}{P}O_2 + \overset{-3}{P}H_3$

(ii) $NaH_2PO_2 + NaOH \longrightarrow \overset{+5}{P}O_4^{3-}$

(i) It depends upon the concentration of NaOH

(ii) The second step of reaction is possible when the concentration of NaOH is high. So the answer of this question can be two.

Answer may be [C] or +1 and –3 because concentration of NaOH is not specified in the question.

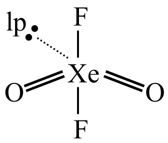
So JEE has given zero marks to all.

Q.25 The shape of XeO_2F_2 molecule is -

(A) trigonal bipyramidal (B) square planar (C) tetrahedral (D) see-saw

Ans. **[D]**

Sol.



see-saw

Q.26 For a dilute solution containing 2.5 g of a non-volatile non-electrolye solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2ºC. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take $K_b = 0.76$ K kg mol^{-1})

(A) 724 (B) 740 (C) 736 (D) 718

Ans. **[A]**

Sol. $\Delta T_b = K_b \times m$

$2 = \dfrac{0.76 \times 2.5/M}{0.1}$

M = 9.5

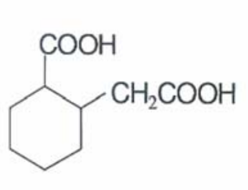
Since solute is present in less amount

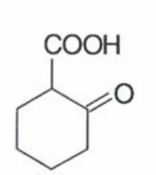
$\dfrac{P^\circ - P_S}{P^\circ} = \dfrac{n}{N}$

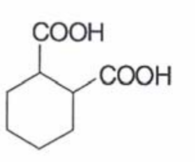
$\dfrac{760 - P_S}{760} = \dfrac{2.5/9.5}{100/18}$

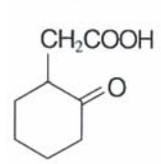
$P_s = 724$

Q.27 The compound that undergoes decarboxylation most readily under mild condition is -

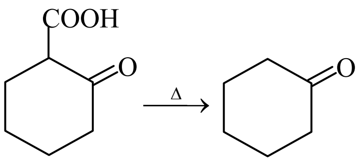
(A) 

(B) 

(C) 

(D) 

Ans. **[B]**

Sol.  $+ CO_2\uparrow$

Q.28 Using the data provided, calculate the multiple bond energy (kJ mol^{-1}) of a C≡C bond in C_2H_2. That energy is (take the bond energy of a C–H bond as 350 kJ mol^{-1})

$2C(s) + H_2(g) \longrightarrow C_2H_2(g)$ $\quad \Delta H = 225$ kJ mol^{-1}

$2C(s) \longrightarrow 2C(g)$ $\quad \Delta H = 1410$ kJ mol^{-1}

$H_2(g) \longrightarrow 2H(g)$ $\quad \Delta H = 330$ kJ mol^{-1}

(A) 1165 (B) 837

(C) 865 (D) 815

Ans. **[D]**

Sol. $\Delta H_R = (B.E)_R - (B.E)_P$

$225 = [1410 + 330] - [-\Delta H_{C\equiv C} + 2 \times 350]$

$\Delta H_{C\equiv C} = 815$ KJ/mol

SECTION – II : Paragraph Type

This section contains 6 **multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

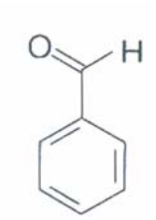
Paragraph for Questions 29 and 30

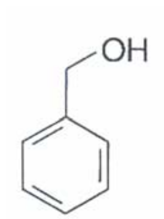
In the following reaction sequence, the compound J is an intermediate.

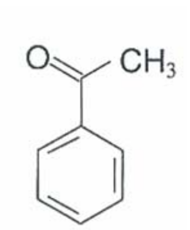
$$I \xrightarrow[CH_3COONa]{(CH_3CO)_2O} J \xrightarrow[\substack{(ii) SOCl_2 \\ (iii) anhyd. AlCl_3}]{(i) H_2 Pd/C} K$$

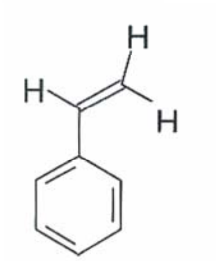
J ($C_9H_8O_2$) gives effervescence on treatment with $NaHCO_3$ and a positive Baeyer's test

Q.29 The compound I is -

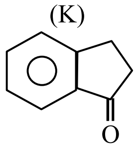
(A) 

(B) 

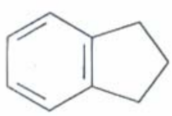
(C) 

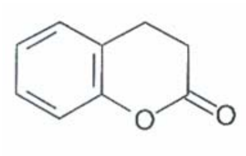
(D) 

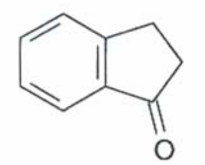
Ans. **[A]**

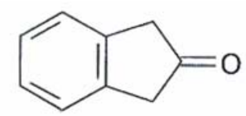
Sol. (I) Ph–CHO $\xrightarrow[CH_3.CO\bar{O}\overset{+}{Na}]{(CH_3.CO)_2O}$ (J) Ph–CH=CH–COOH $\xrightarrow[\substack{(2) SOCl_2 \\ (3) Anhy. AlCl_3}]{(1) H_2, Pd/C}$ (K) 

Q.30 The compound K is -

(A) 

(B) 

(C) 

(D) 

Ans. **[C]**

Paragraph for Questions 31 and 32

The electrochemical cell shown below is a concentration cell.

$M|\ M^{2+}$ (saturated solution of a sparingly soluble salt, MX_2). || M^{2+} (0.001 mol dm^{-3}) | M

The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059 V.

Q.31 The value of ΔG(kJ mol^{-1}) for the given cell is (take 1F = 96500 C mol^{-1}) :

(A) – 5.7 (B) 5.7 (C) 11.4 (D) –11.4

Ans. **[D]**

Sol. $\Delta G^\circ = -\ nFE^\circ_{cell}$

$= -2 \times 96500 \times \dfrac{0.059}{1000}$

$= -11.387$

$\cong -11.4$

Q.32 The solubility product (K_{sp}; $mol^3\ dm^{-9}$) of MX_2 at 298 K based on the information available for the given concentration cell is (take 2.303 × R × 298/F = 0.059 V)

(A) 1×10^{-15} (B) 4×10^{-15} (C) 1×10^{-12} (D) 4×10^{-12}

Ans. **[B]**

Sol. $E_{cell} = 0 - \dfrac{.0591}{2} \log \dfrac{M^{+2}}{0.001}$

$M^{+2} = 1 \times 10^{-5}$

$MX_2 \rightleftharpoons M^{+2} + 2X^-$

S 2S

$K_{sp} = S(2S)^2$

$= 4S^3$

$= 4 \times (10^{-5})^3$

$= 4 \times 10^{-15}$

Paragraph for Questions 33 and 34

Bleaching powder and bleach solution are produced on a large scale and used in several house-hold products. The effectiveness of bleach solution is often measured by iodometry.

Q.33 Bleaching power contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

(A) Cl_2O (B) Cl_2O_7 (C) ClO_2 (D) Cl_2O_6

Ans. **[A]**

Sol. $2HOCl \longrightarrow Cl_2O + H_2O$

Q.34 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is -

(A) 0.48 M (B) 0.96 M (C) 0.24 M (D) 0.024 M

Ans. **[C]**

Sol.

–2

2

0 1

$$CaOCl_2 + 2KI + 2CH_3COOH \rightarrow CaCl_2 + I_2 + 2CH_3COOK + H_2O$$

n = 2 n = 1

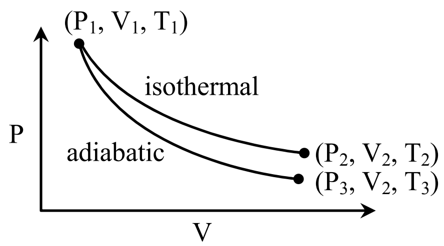
$\frac{48 \times 0.25}{2}$ $M = \frac{48 \times 0.25}{2 \times 25} = 0.24$ $\frac{48 \times 0.25}{2}$

= 0.24

Section III : Multiple Correct Answer(s) Type

This section contains 6 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE** is correct.

Q.35 The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct ?



(A) $T_1 = T_2$ (B) $T_3 > T_1$ (C) $w_{isothermal} > w_{adabatic}$ (D) $\Delta U_{isothermal} > \Delta U_{adiabatic}$

Ans. **[A,D]**

Q.36 For the given aqueous reactions, which of the statement(s) is (are) true ?

$$\text{excess } KI + K_3[Fe(CN)_6] \xrightarrow{\text{dil. } H_2SO_4} \text{brownish-yellow solution}$$

$\downarrow ZnSO_4$

white precipitate + brownish-yellow filtrate

$\downarrow Na_2S_2O_3$ (brownish-yellow filtrate)

colourless solution

(A) The first reaction is a redox reaction

(B) White precipitate is $Zn_3[Fe(CN)_6]_2$

(C) Addition of filtrate to starch solution gives blue colour

(D) White precipitate is soluble in NaOH solution

Ans. **[A,C,D]**

Sol. $KI + K_3Fe(CN)_6 \longrightarrow K_4Fe(CN)_6 + KI_3$
Brownish yellow

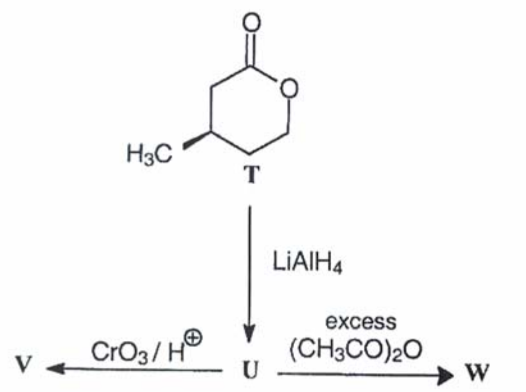
$KI_3 \xrightarrow{ZnSO_4}$ No reaction

$K_4[Fe(CN)_6] + ZnSO_4 \longrightarrow K_2Zn_3[Fe(CN)_6]_2$
white ppt.

$KI_3 + Na_2SO_3 \longrightarrow NaI + KI + Na_2S_4O_6$
starch blue

$K_2Zn_3[Fe(CN)_6]_2 + NaOH$
$\downarrow$
$[Fe(CN)_6]^{4-} + [Zn(OH)_4]^{2-}$

Q.37 With reference to the scheme given, which of the given statement(s) about **T, U, V** and **W** is (are) correct ?



(A) **T** is soluble in hot aqueous NaOH

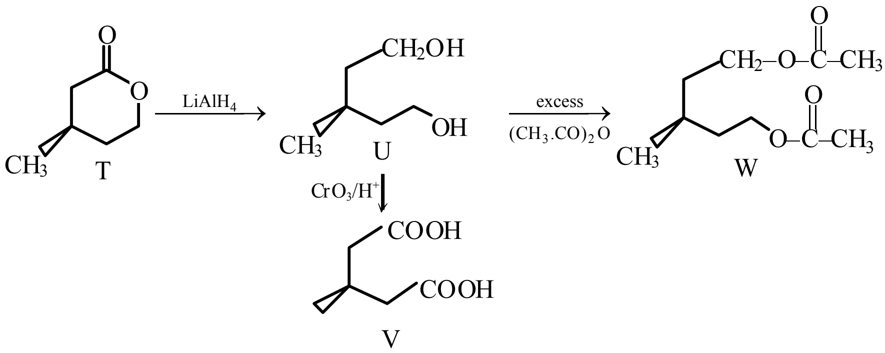
(B) **U** is optically active

(C) **M**olecular formula of **W** is $C_{10}H_{18}O_4$

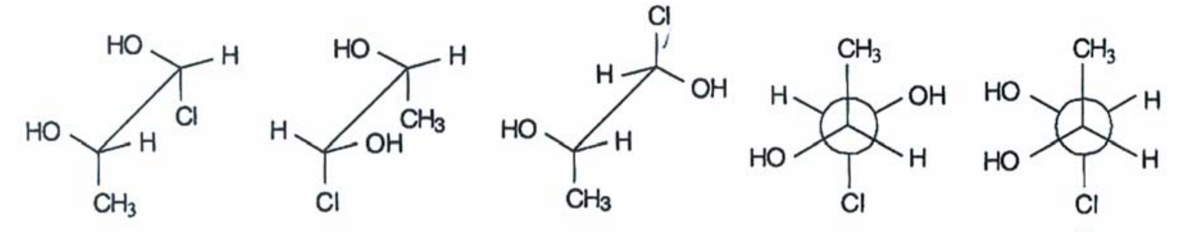
(D) **V** gives of effervescence on treatment with aqueous $NaHCO_3$

Ans. **[A,C,D]**

Sol.



Q.38 Which of the given statement(s) about **N**, **O**, **P** and **Q** with respect to **M** is (are) correct ?



(A) **M** and **N** are non-mirror image stereoisomers

(B) **M** and **O** are identical

(C) **M** and **P** are enantiomers

(D) **M** and **Q** are identical

Ans. **[A,B,C]**

Q.39 With respect to graphite and diamond, which of the statement(s) given below is (are) correct ?

(A) Graphite is harder than diamond

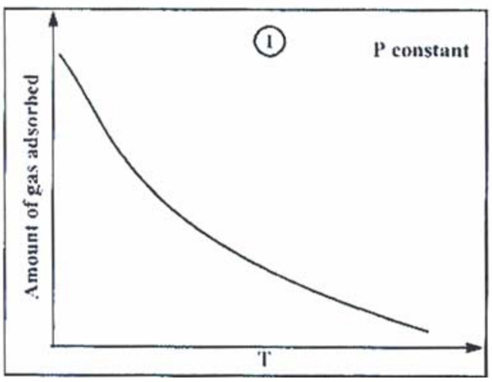
(B) Graphite has higher electrical conductivity than diamond

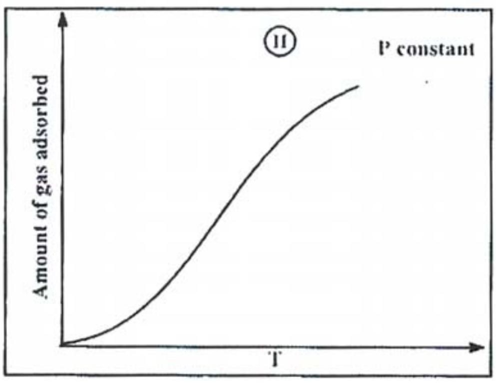
(C) Graphite has higher thermal conductivity than diamond

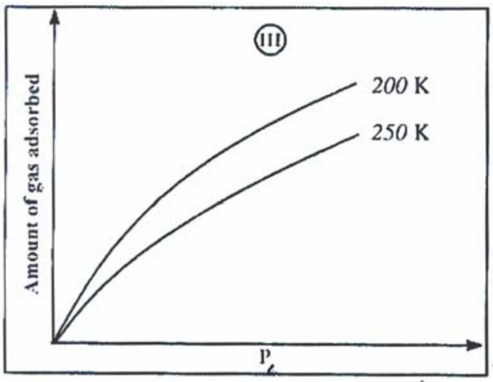
(D) Graphite has higher C–C bond order than diamond.

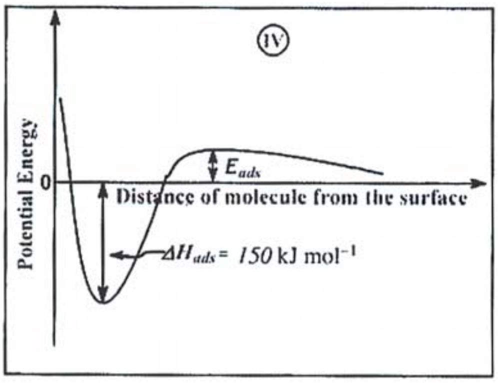
Ans. **[B,D]**

Q.40 The given graphs/data **I**, **II**, **III** and **IV** represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about **I**, **II**, **III** and **IV** is (are) correct ?









(A) **I** is physisorption and **II** is chemisorption
(B) **I** is physisorption and **III** is chemisorption
(C) **IV** is chemisorption and **II** is chemisorption
(D) **IV** is chemisorption and **III** is chemisorption

Ans. **[A,C]**

Sol. I physiosorption

II chemisorption

III physiosorption

IV chemisorption

Part – III : (Mathematics)

SECTION – I (Single Correct Answer Type)

This section contains 8 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Q.41 Let $a_1, a_2, a_3, \ldots\ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D) 25

Ans. **[D]**

Sol. $a_1 = 5 \qquad a_{20} = 25$

$T_1 = \frac{1}{5} \qquad T_{20} = \frac{1}{25}$

$T_{20} = \frac{1}{5} + 19D = \frac{1}{25}$

$D = \left(\frac{1}{25} - \frac{1}{5}\right)\frac{1}{19}$

$= -\frac{20}{5(25)(19)}$

$T_n = \frac{1}{5} - \frac{(n-1)(20)}{(125)(19)}$

$= \frac{(25)(19) - (n-1)(20)}{(125)(19)} < 0$

$(25)\,(19) < (n-1)\,(20)$

$n - 1 > \frac{(25)(19)}{(20)}$

$n > \frac{5(19)}{4} + 1$

$n > \frac{95}{4} + 1$

$n > 23.75 + 1$

$n > 24.75$

$n = 25$

Q.42 The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, – 1) is

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

Ans. **[A]**

Sol. Equation of plane passing through intersecting of plane P_1 & P_2

is $P_1 + \lambda P_2 = 0$

$(1 + \lambda) x + (2 - \lambda) y + (3 + \lambda) z - 2 - 3\lambda = 0$

distance of plane from pt (3, 1, –1) is $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{|3 + 3\lambda + 2 - \lambda - 3 - \lambda - 2 - 3\lambda|}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3-\lambda)^2}}$$

on solving $\lambda = -\frac{7}{2}$

so equation of plane is

$$\left(1 - \frac{7}{2}\right)x + \left(2 + \frac{7}{2}\right)y + \left(3 - \frac{7}{2}\right)z - 2 + \frac{21}{2} = 0$$

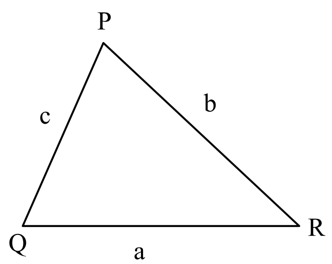
$5x - 11y + z = 17$

Q.43 Let PQR be a triangle of area Δ with a = 2, b = $\frac{7}{2}$ and c = $\frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Ans. **[C]**

Sol.



$$\left.\begin{aligned} a &= 2 \\ b &= 7/2 \\ c &= 5/2 \end{aligned}\right\} \Rightarrow s = 4$$

$$\frac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \frac{1-\cos P}{1+\cos P} \qquad \Delta = \sqrt{4.2.\frac{1}{2}.\frac{3}{2}} = \sqrt{6}$$

$$= \tan^2\frac{P}{2}$$

$$= \left(\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right)^2$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{4(2)} = \frac{3}{32}$$

$$= \left(\frac{9}{16.6}\right) = \left(\frac{3}{4\Delta}\right)^2$$

Q.44 If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

Ans. **[C]**

Sol. $|\vec{a}+\vec{b}| = \sqrt{29}$

$\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$

$(\vec{a}+\vec{b}) \times (2\hat{i}+3\hat{j}+4\hat{k}) = \vec{0}$

$\vec{a}+\vec{b} = \lambda\ (2\hat{i}+3\hat{j}+4\hat{k})$

$|\vec{a}+\vec{b}| = \sqrt{4\lambda^2+9\lambda^2+16\lambda^2} = |\lambda|\ \sqrt{29}$

$\Rightarrow \lambda = 1, -1$

$\vec{a}+\vec{b} = \pm(2\hat{i}+3\hat{j}+4\hat{k})$

$(\vec{a}+\vec{b}) . (-7\hat{i}+2\hat{j}+3\hat{k})$

$= \pm(2\hat{i}+3\hat{j}+4\hat{k}).(-7\hat{i}+2\hat{j}+3\hat{k}) = \pm 4$

Q.45 If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$ (C) $PX = 2X$ (D) $PX = -X$

Ans. **[D]**

Sol. $P^T = 2P + I$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$

$\Rightarrow P = -I$

$PX = -I X$

$PX = -X$

Q.46 Let α (a) and β(a) be the roots of the equation $\left(\sqrt[3]{1+a} - 1\right)x^2 + \left(\sqrt{1+a} - 1\right)x + \left(\sqrt[6]{1+a} - 1\right) = 0$ where $a > -1$.

Then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$ are

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

Ans. **[B]**

Sol. $(1 + a) = t^6$

$(t^2 - 1)x^2 + (t^3 - 1)x + (t - 1) = 0$

$$x = \frac{-(t^3 - 1) \pm \sqrt{(t^3 - 1)^2 - 4(t - 1)(t^2 - 1)}}{2(t^2 - 1)}$$

$$x = \frac{-(t^3 - 1) \pm (t - 1)\sqrt{(t^2 + t + 1)^2 - 4(t + 1)}}{2(t - 1)(t + 1)}$$

$$x = \frac{-(t^2 + t + 1) \pm \sqrt{(t^2 + t + 1)^2 - 4(t + 1)}}{2(t + 1)}$$

$a \to 0^+ \Rightarrow t \to 1^+$

$$x = \frac{-3 \pm \sqrt{9 - 8}}{2(2)} \Rightarrow x = \frac{-3 \pm 1}{4}$$

$$\Rightarrow x = -1, -\frac{1}{2}$$

Q.47 Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Ans. **[A]**

Sol. required probability = 1 – P(D_4 has diff.)

$$= 1 - \left(\frac{6.1.1.5 + {}^3C_2.6.1.5.4 + 6.5.4.3}{6^4}\right)$$

$$= \frac{91}{216}$$

Q.48 The value of the integral

$$\int_{-\pi/2}^{\pi/2}\left(x^2 + \ln\frac{\pi + x}{\pi - x}\right)\cos x\, dx \text{ is}$$

(A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

Ans. **[B]**

Sol. $\int_{-\pi/2}^{\pi/2} x^2 \cos x\, dx + \int_{-\pi/2}^{\pi/2} \ell n\left(\frac{\pi - x}{\pi + x}\right)\cos x\, dx$

↓ Even function ↓ Odd function

$$= 2\int_0^{\pi/2} x^2 \cos x\, dx + 0$$

$$= 2[x^2 \sin x + 2x\cos x - 2\sin x]_0^{\pi/2} = 2\left[\frac{\pi^2}{4} - 2\right]$$

$$= \frac{\pi^2}{2} - 4$$

SECTION II : Paragraph Type

This section contains 6 **multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 49 and 50

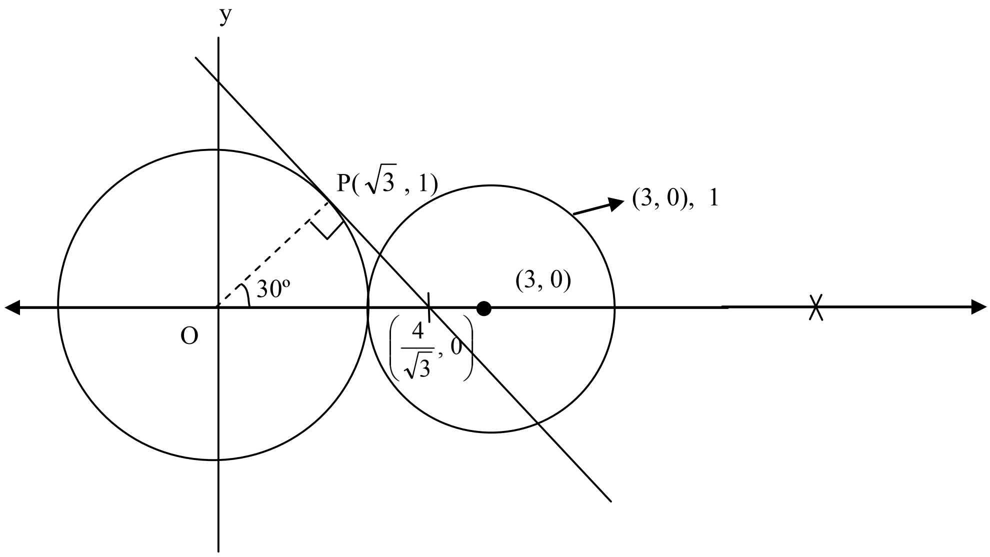
A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3}, 1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

Q.49 A possible equation of L is

(A) $x - \sqrt{3}\,y = 1$ (B) $x + \sqrt{3}\,y = 1$ (C) $x - \sqrt{3}\,y = -1$ (D) $x + \sqrt{3}\,y = 5$

Ans. **[A]**

Sol.



Slope of PT = tan (120º) = $-\sqrt{3}$

Slope of line L = $\frac{1}{\sqrt{3}}$

Line L ≡ $x - \sqrt{3}\,y + \lambda = 0$

tangent to $(x-3)^2 + y^2 = 1$

$$\frac{|3+\lambda|}{2} = 1$$

$\lambda + 3 = 2, -2$

$\lambda = -1, -5$

$x - \sqrt{3}\,y - 1 = 0$

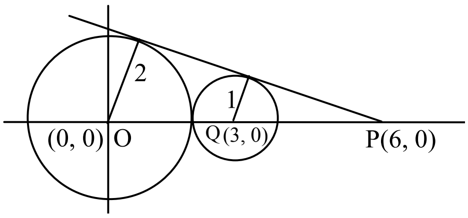
or $x - \sqrt{3}\,y - 5 = 0$

Q.50 A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}\,y = 4$ (D) $x + 2\sqrt{2}\,y = 6$

Ans. **[D]**

Sol. Common tangent both circles



So $P \equiv (6, 0)$

line through P

$\lambda x - y - 6\lambda = 0$

tangent to circle $\frac{|3\lambda|}{\sqrt{1+\lambda^2}} = 1$

$9\lambda^2 = 1 + \lambda^2 \Rightarrow \lambda^2 = \frac{1}{8}$

$\lambda = \frac{1}{2\sqrt{2}}, \frac{-1}{2\sqrt{2}}$

Equation of tangent $x + 2\sqrt{2}\, y = 6$

Paragraph for Questions 51 and 52

Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all $x \in IR$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\, dt$ for all $x \in (1, \infty)$.

Q.51. Consider the statements :

P : There exists some $x \in IR$ such that $f(x) + 2x = 2(1 + x^2)$

Q : There exists some $x \in IR$ such that $2f(x) + 1 = 2x(1 + x)$

Then

(A) both **P** and **Q** are true

(B) **P** is true and **Q** is false

(C) **P** is false and **Q** is true

(D) both **P** and **Q** are false

Ans. **[C]**

Sol. (P): $(\sin^2 x)(1 - x)^2 + x^2 + 2x = 2 + 2x^2$

$(\sin^2 x)(1 - x)^2 = x^2 - 2x + 2$

$\sin^2 x = \frac{(1-x)^2 + 1}{(1-x)^2}$, which is greater than 1 $\Rightarrow$ no solution

$\Rightarrow$ P is false.

(Q) : $2\sin^2 x = \frac{x^2}{(1-x)^2} - 1$

$0 \le \frac{x^2}{(1-x)^2} - 1 \le 2$.

This inequality is satisfied by some value of x.

Q.52 Which of the following is true?

(A) g is increasing on $(1, \infty)$

(B) g is decreasing on $(1, \infty)$

(C) g is increasing on (1, 2) and decreasing on $(2, \infty)$

(D) g is decreasing on (1, 2) and increasing on $(2, \infty)$

Ans. **[B]**

Sol. $$g'(x) = \underbrace{\left(\frac{2(x-1)}{x+1} - \ell n\, x\right)}_{\substack{-ve \\ \text{for } x>1}} \underbrace{f(x)}_{\text{always}+ve}$$

$g'(x) < 0 \Rightarrow g(x) \downarrow$ on $(1, \infty)$

SECTION – I (Single Correct Answer Type)

Paragraph for Questions 53 to 54

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

Q.53 The value of b_6 is

(A) 7 (B) 8

(C) 9 (D) 11

Ans. **[B]**

Sol. for b_n

first and last place are fixed by 1

so case (1) if only one zero is used such cases = ${}^{n-2}C_1$

case (2) if two zero are used the two zeros are such that no two zeros are consecutive = ${}^{n-3}C_2$

case (3) if three zeros are used then the positing of three zeros such that no two zeros are consecutive $= {}^{n-4}C_3$

So $b_n = {}^{n-2}C_1 + {}^{n-.3}C_2 + {}^{n-4}C_3 + {}^{n-5}C_4 + {}^{n-6}C_5$ ....

for $b_6 = {}^4C_1 + {}^3C_2 + 1$

when no zero is used

= 8

Q.54 Which of the following is correct?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$

(C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

Ans. **[A]**

Sol. $b_n = 1 + {}^{n-2}C_1 + {}^{n-3}C_2 + {}^{n-4}C_3 + {}^{n-5}C_4 + {}^{n-6}C_5 + \ldots\ldots$

$c_n = 1 + {}^{n-3}C_1 + {}^{n-4}C_2 + {}^{n-5}C_3 + {}^{n-6}C_4 + \ldots\ldots$

$a_n = 1 + {}^{n-1}C_1 + {}^{n-2}C_2 + {}^{n-3}C_3 + {}^{n-4}C_4 + \ldots\ldots$

$a_{17} = 1 + {}^{16}C_1 + {}^{15}C_2 + {}^{14}C_3 + {}^{13}C_4 + {}^{12}C_5 + {}^{11}C_6 + {}^{10}C_7 + {}^{9}C_8 + {}^{8}C_9$

$a_{16} = 1 + {}^{15}C_1 + {}^{14}C_2 + {}^{13}C_3 + {}^{12}C_4 + {}^{11}C_5 + {}^{10}C_6 + {}^{9}C_7 + {}^{8}C_8$

$a_{15} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

$a_{17} = a_{16} + a_{15}$ so A is correct.

$c_{17} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7 + {}^{7}C_8$

$c_{16} = 1 + {}^{13}C_1 + {}^{12}C_2 + {}^{11}C_3 + {}^{10}C_4 + {}^{9}C_5 + {}^{8}C_6 + {}^{7}C_7$

$c_{15} = 1 + {}^{12}C_1 + {}^{11}C_2 + {}^{10}C_3 + {}^{9}C_4 + {}^{8}C_5 + {}^{7}C_6$

$c_{17} = c_{16} + c_{15}$

So, B is wrong.

$b_{17} = 1 + {}^{15}C_1 + {}^{14}C_2 + {}^{13}C_3 + {}^{12}C_4 + {}^{11}C_5 + {}^{10}C_6 + {}^{9}C_7 + {}^{8}C_8$

$b_{16} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

$c_{16} = 1 + {}^{13}C_1 + {}^{12}C_2 + {}^{11}C_3 + {}^{10}C_4 + {}^{9}C_5 + {}^{8}C_6 + {}^{7}C_7$

$b_{17} = b_{16} + c_{17}$ so C is wrong.

$a_{17} = 1 + {}^{16}C_1 + {}^{15}C_2 + {}^{14}C_3 + {}^{13}C_4 + {}^{12}C_5 + {}^{11}C_6 + {}^{10}C_7 + {}^{9}C_8 + {}^{8}C_7$

$c_{17} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

$b_{16} = 1 + {}^{14}C_1 + {}^{13}C_2 + {}^{12}C_3 + {}^{11}C_4 + {}^{10}C_5 + {}^{9}C_6 + {}^{8}C_7$

so D is wrong.

SECTION III : Multiple Correct Answer(s) Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

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55. For every integer n, let a_n and b_n be real numbers. Let function f: IR → IR be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}, \text{ for all integers n.}$$

If f is continuous, then which of the following hold(s) for all n?

(A) $a_{n-1} - b_{n-1} = 0$

(B) $a_n - b_n = 1$

(C) $a_n - b_{n+1} = 1$

(D) $a_{n-1} - b_n = -1$

Ans. **[B, D]**

Sol. At $x = 2n$

$x \to 2n^+$ $\quad a_n + \sin 2n\pi = a_n$

$x \to 2n^-$ $\quad b_n + \cos 2n\pi = b_n + 1$

For continuous $\quad a_n = b_n + 1$

At $x = 2n + 1$

$x \to 2n + 1^+$ $b_{n+1} + \cos\pi(2n + 1) = b_{n+1} - 1$
$x \to 2n + 1^-$ $a_n + \sin \pi(2n + 1) = a_n$
for continuous $a_n = b_{n+1} - 1$
$a_n - b_{n+1} = -1$
for $n = n - 1$ $a_{n-1} - b_n = -1$

Q.56 If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is (are)

(A) $y + 2z = -1$ (B) $y + z = -1$
(C) $y - z = -1$ (D) $y - 2z = -1$

Ans. **[B, C]**

Sol. If these two lines are coplanar then shortest distance between them = 0

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0$$

$k = 2$ or -2

so lines are $\left.\begin{array}{l} \frac{x-1}{2} = \frac{y+1}{2} = \frac{z}{2} \\ \text{and } \frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{2} \end{array}\right\}$ set (i)

OR

$\left.\begin{array}{l} \frac{x-1}{2} = \frac{y+1}{-2} = \frac{z}{2} \\ \text{and } \frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{-2} \end{array}\right\}$ set (ii)

the plane which contain these set of line should contain the points (1, –1, 0) and (–1, –1, 0) which is satisfied by all the four options and $(2\hat{i} + 2\hat{j} + 2\hat{k})$ & $(5\hat{i} + 2\hat{j} + 2\hat{k})$ OR

$(2\hat{i} - 2\hat{j} + 2\hat{k})$ & $(5\hat{i} + 2\hat{j} - 2\hat{k})$ are perpendicular to normal of plane

For first set option (C) is correct.

For second set option (B) is correct.

Q.57 If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

(A) –2 (B) –1 (C) 1 (D) 2

Ans. **[A, D]**

Sol. adj (P) = $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$|\text{adj } P| = |P|^{n-1} = |P|^2$

$|\text{adj } P| = 4 = |P|^2$

$|P| = 2$ or -2

Q.58 Let f : (–1, 1) → IR be such that f(cos 4θ) = $\frac{2}{2-\sec^2\theta}$ for θ ∈ $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

(A) $1-\sqrt{\frac{3}{2}}$ (B) $1+\sqrt{\frac{3}{2}}$ (C) $1-\sqrt{\frac{2}{3}}$ (D) $1+\sqrt{\frac{2}{3}}$

Ans. **[A, B]**

Sol. $\cos 4\theta = \frac{1}{3}$

$2\cos^2 2\theta - 1 = \frac{1}{3}$

$\cos 2\theta = \pm\sqrt{\frac{2}{3}}$

On solving

$\cos^2\theta = \frac{1 \pm \sqrt{2/3}}{2}$

$\sec^2\theta = \frac{2}{1 \pm \sqrt{2/3}}$

On putting this

$f\left(\frac{1}{3}\right) = \pm\sqrt{\frac{3}{2}} + 1$

Note : As f(1/3) is having two values, so this relation is not a function. So JEE has given zero marks to all.

Q.59 Let X and Y be two events such that P(X|Y) = $\frac{1}{2}$, P(Y|X) = $\frac{1}{3}$ and P(X ∩ Y) = $\frac{1}{6}$. Which of the following is (are) correct?

(A) P(X ∪ Y) = $\frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^C \cap Y) = \frac{1}{3}$

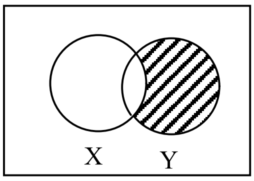
Ans. **[A, B]**

$P\left(\frac{X}{Y}\right) = \frac{P(X\cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$\frac{P(Y\cap X)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

P(X ∩ Y) = P(X).P(Y) True



$(X^C \cap Y)$

$P(X^C \cap Y) = P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

Q.60 If $f(x) = \int_0^x e^{t^2} (t-2)(t-3)\,dt$ for all $x \in (0, \infty)$, then

(A) f has a local maximum at x = 2

(B) f is decreasing on (2, 3)

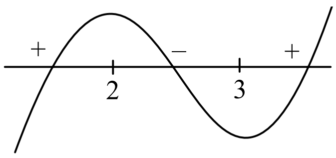
(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) f has a local minimum at x = 3

Ans. **[A, B, C, D]**

Sol. $f(x) = \int_0^x e^{t^2} (t-2)(t-3)\,dt$

$f'(x) = e^{x^2} (x-2)(x-3)$



$f'(x) < 0 \quad \forall x \in (2, 3)$

so f(x) is decreasing on (2, 3)

also at x = 2, $f'(x)$ changes its sign from +ve to –ve.

Hence x = 2 is point of maxima

At x = 3, $f'(x)$ changes its sign from –ve to +ve.

Hence x = 3 is point of minima.

Also $f'(2) = f'(3) = 0$

So from Rolle's Theorem there exist a point c such that $f''(c) = 0$