THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test – Kaprekar Contest
NMTC at SUB JUNIOR LEVEL – VII to VIII Standards
Saturday, 26th August 2017

Notes:
1. Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.
2. Diagrams are only visual aids; they are NOT drawn to scale.
3. You are free to do rough work on separate sheets.
4. Duration of the test: 2 pm to 4 pm – 2 hours.

PART - A

Note
• Only one of the choices A, B, C, D is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering, seek the guidance of the supervisor.

• For each correct response you get 1 mark. For each incorrect response you lose 1/2 mark.

Q.1 The fraction $\frac{4}{37}$ is written in the decimal form $0.a_1a_2a_3 \ldots$ The value of $a_{2017}$ is:
A. 8  B. 0  C. 1  D. 5

Sol. Option (C)

$\frac{4}{37} = 0. a_1a_2a_3$

$\therefore \frac{4}{37} = 0.108108108 = 0.\overline{108}$

$\therefore a_1 = 1, a_2 = 0, a_3 = 8$

$a_4 = 1, a_5 = 0, \ldots$ So on

$2017 \div 3 \Rightarrow$ Remainder = 1

$\therefore a_{2017} = 1$

Q.2 The number of integers x satisfying the equation $(x^2 - 3x + 1)^{x+1} = 1$ is:
A. 2  B. 3  C. 4  D. 5

Sol. Option (C)
\[(x^2 - 3x + 1)^{x+1} = 1\]

**Case (I):**
\[x^2 - 3x + 1 = 1\]
\[x^2 - 3x = 0\]
\[x(x - 3) = 0\]
\[x = 0, \ x = 3.\]

**Case (II):**
\[x + 1 = 0\]
\[x = -1\]

**Case (III):**
When
\[x^2 - 3x + 1 = -1\]
\[x^2 - 3x + 2 = 0\]
\[x^2 - 2x - x + 2 = 0\]
\[x(x - 2) - 1(x - 2) = 0\]
\[(x - 1)(x - 2) = 0\]
\[x = 1, \ x = 2\]

- Put \(x = 1\)
- LHS \((1 - 3 + 1)^2\)
- \((-1)^2 = 1 = \text{RHS}\)
- \(x = 1\)
- Put \(x = 2\)
- LHS \((4 - 6 + 1)^3\)
- \((-1)^3 \neq 1\)
- \(\Rightarrow x = 2\) not possible

\[\text{no. of integers are 4}\]

**Q.3** The number of two digit numbers \(ab\) such that the number \(ab - ba\) is prime number is:
A. 0   B. 1   C. 2   D. 3

**Sol.** Option (A)

- \(ab\) is a two digit no then \((ab - ba)\) is always multiple of 9.
- \(\Rightarrow\) No. of prime no = 0.

**Q.4** If \(A = \frac{5425}{1444} - \frac{3045}{2987} - \frac{493}{4284}\), then

A. 1 < A < 2   B. 2 < A < 3   C. 3 < A < 4   D. A < 1

**Sol.** Option (B)

- If \(A = \frac{5425}{1444} - \frac{3045}{2987} - \frac{493}{4284}\)
- \(= 3.756 - 0.980 - 0.115\)
- \(= 2.66\)
- \(\Rightarrow 2 < A < 3\)

**Q.5** What is the 2017th letter in ABRACADABRAABRACADABRA ..., where the word ABRACADABRA is repeatedly written?
A. A   B. B   C. C   D. R

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Sol. Option (A)

\[ \therefore \text{ ABRACADABRA is repeatedly written} \]
\[ \Rightarrow \text{ after 11 letters, repetition is there} \]
\[ \frac{2017}{11} \rightarrow \text{Remainder is 4} \]
\[ \therefore 2017^{th} \text{ letter is A} \]

Q.6 How many of the following statements are true?
(a) A 10% increase followed by another 5% increase is equivalent to a 15% increase
(b) If the radius of a circle is doubled then the ratio of the area of the circle to the circumference is doubled
(c) If a positive fraction is subtracted from 1 and the resulting fraction is again subtracted from 1 we get the original fraction.

A. 0 \hspace{1cm} B. 1 \hspace{1cm} C. 2 \hspace{1cm} D. 3

Sol. Option (C)

(a) False

Equivalent increase
\[ = 10 + 5 - \frac{10 \times 5}{100} \]
\[ = 15 - 0.5 \]
\[ = 14.5\% \]
\[ \Rightarrow \text{False} \]

(b) \[ C_1 = 2\pi r \]
\[ A_1 = \pi r^2 \]
\[ \frac{A_1}{C_1} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} \]

when \( r \rightarrow 2r \)

\[ C_2 = 4\pi r \]
\[ A_2 = 4\pi r^2 \]
\[ \frac{A_2}{C_2} = r \]
\[ \therefore \frac{A_2}{C_2} = 2 \left( \frac{A_1}{C_1} \right) \]

True

(C) Let positive fraction is \( \frac{a}{b} \)

it is subtract from 1.
\[
\left(1 - \frac{a}{b}\right) = \frac{b-a}{b}
\]

when \(\frac{b-a}{b}\) is subtract from 1

\[
1 - \left(\frac{b-a}{b}\right) = \frac{a}{b} \Rightarrow \text{Original fraction}
\]

True

\(\Rightarrow \) No. of statements True = 2

Q.7 In the adjoining figure the breadth of the rectangle is 10 units. Two semicircles are drawn on the breadth as diameter. The area of the shaded region is 100 sq units. The shortest distance between the semicircles is:

\[
\text{A. } \frac{5\pi}{2} \quad \text{B. } 5\pi \quad \text{C. } \frac{5\pi}{3} \quad \text{D. } \frac{3\pi}{4}
\]

Sol. Option (A)

Area of shaded region = 100 sq. unit.
Shortest distance is PQ between semi-circle.
Let AB = x

\[
\therefore \text{area of rectangle} = \text{area of two semi-circle} + \text{Area of shaded region}
\]

\[
x \times 10 = 2 \times \frac{\pi}{2} (5)^2 + 100
\]

\[
10x = 25\pi + 100
\]

\[
x = \frac{25\pi + 100}{10} = \frac{5\pi}{2} + 10
\]

shortest distance PQ

\[
= x - (QC_2 + PC_1)
\]

\[
x = 10
\]

\[
= \frac{5\pi}{2} + 10 - 10
\]

\[
= \frac{5\pi}{2}
\]
Q.8 When you arrange the following in descending order:
A. 15% of 30
B. 8% of 15
C. 20% of 20
D. 26% of 10
E. 9% of 25
The middle one is:
A. 15% of 30  B. 8% of 15  C. 20% of 20  D. 26% of 10

Sol. Option (D)

(A) 15% of 30 = \(\frac{15}{100} \times 30 = 4.5\)

(B) 8% of 15 = \(\frac{8}{100} \times 15 = 1.2\)

(C) 20% of 20 = \(\frac{20}{100} \times 20 = 4\)

(D) 26% of 10 = \(\frac{26}{100} \times 10 = 2.6\)

(E) 9% of 25 = \(\frac{9}{100} \times 25 = 2.25\)

Descending order
4.5, 4, 2.6, 2.25, 1.2
Middle term = 2.6
⇒ 26% of 10

Q.9 A boy aims at a target shown in the figure. When he hits the center circle he gets 7 points, first annular region 5 points and second annular region 3 points. He shoots six times. Which one of the following is a possible score?

![Target Diagram]

A. 16  B. 26  C. 19  D. 41

Sol. Option (B)

Possible score
Hits center circle → 1 time
First annular region → 2 times

Second annular region → 3 times
∴ Possible score
= 7 + (5 × 2) + (3 × 3) = 7 + 10 + 9
= 26

Q.10 After simplifying the fraction
\[
\begin{align*}
\left(\frac{a + b - a}{1 + ab}\right) & \quad \left(\frac{x + y - y}{1 - xy}\right) \\
\left(\frac{1}{1 + ab}\right) & \quad \left(\frac{1 + y(x + y)}{1 - xy}\right)
\end{align*}
\]
we get a term independent of
A. a, y  \quad B. b, x  \quad C. a, b  \quad D. x, y

Sol. Option (A)
\[
\begin{align*}
\frac{a + b - a}{1 + ab} & \quad \frac{x + y - y}{1 - xy} \\
\frac{1}{1 + ab} & \quad \frac{1 + y(x + y)}{1 - xy}
\end{align*}
\]
\[
\begin{align*}
\left(\frac{a + a^2 b + b - a}{1 + ab}\right) & \quad \left(\frac{x + y - y + xy^2}{1 - xy}\right) \\
\left(\frac{1 + ab - ab + a^2}{1 + ab}\right) & \quad \left(\frac{1 - xy + xy + y^2}{1 - xy}\right)
\end{align*}
\]
\[
\begin{align*}
& = \frac{b(1 + a^2)}{(1 + a^2)} \quad \frac{x(1 + y^2)}{(1 + y^2)} \\
& = bx
\end{align*}
\]
which is independent of a, y.

Q.11 If 7 Rasagullas are distributed to each boy of a group, 10 rasagullas would be left. If 8 are given to each boy then 5 rasagullas would be left. So the person who distributes the rasagullas brought 15 more rasagullas and distributed the same number (x) rasagullas to each. There is no rasagulla left. Then x is :
A. 10  \quad B. 11  \quad C. 12  \quad D. 14

Sol. Option (C)
Let no. of boys are 'y' & no of rasagullas = r
According to question
Total no. of rasagullas in first case
r = 7y + 10 \quad ...(1)
Second case r = 8y + 5 \quad ...(2)
From (1) & (2)
7y + 10 = 8y + 5
y = 5
∴ No. of boys = y = 5
No. of rasagullas
r = 7(5) + 10 = 45
If 15 more rasagullas are there
∴ total no. of rasagullas
45 + 15 = 60
Now 60 rasagullas are distributed to 5 boys
∴ Each boy get rasagullas
\[ \frac{60}{5} = 12 \]
⇒ \( x = 12 \) Ans.

Q.12 In the adjoining diagram all squares are of the same size. The total area of the figure is 288 square cms. The perimeter of the figure (in cm) is:

\[ \text{A. 86} \quad \text{B. 96} \quad \text{C. 106} \quad \text{D. 92} \]

Sol. Option (B)
No. of squares = 8
Area of 8 square
\[ 8a^2 = 288 \]
a\(^2\) = 36
a = 6
Side of each square = 6 cm

∴ Perimeter
\[ = 16 \times 6 \]
\[ = 96 \text{ cm} \]

Q.13 When Newton was a primary school student he had to multiply a number by 5. But by mistake he divided the number by 5. The percentage error he committed is:

A. 95% \quad \text{B. 96%} \quad \text{C. 50%} \quad \text{D. 75%}

Sol. Option (B)
let no. be \( x \)
multiply no by 5 = 5\( x \)
and obtained result = $\frac{x}{5}$

\[
\text{% error} = \frac{5x - \frac{x}{5}}{5x} \times 100
\]

\[
= \frac{24x}{5x} \times 100
\]

\[
= 24 \times 4
\]

\[
= 96\%
\]

**Q.14**

ABC is an isosceles triangle with sides $AB = AC = 3x - 4 = \frac{3}{4}x + 32$. The area of the equilateral triangle with side length $x$ is:

A. $32\sqrt{3}$

B. $36\sqrt{3}$

C. $54\sqrt{3}$

D. $64\sqrt{3}$

**Sol.**

Option (D)

$AB = AC$

$3x - 4 = \frac{3}{4}x + 32$

$12x - 16 = 3x + 128$

$9x = 144$

$x = 16$

\[\text{Area of equilateral } \Delta ABC = \frac{\sqrt{3}}{4} \times (16)^2 = 64\sqrt{3}\]

**Q.15**

Two distinct numbers $a$ and $b$ are selected from 1, 2, 3, ..., 60. The maximum value of $\frac{a \times b}{a - b}$ is

A. 6750

B. 5270

C. 4850

D. 3540

**Sol.**

Option (D)

Two no. $a$, $b$ selected from 1, 2, 3, ..., 60

\[\text{max. value of } \frac{a \times b}{a - b} = \frac{60 \times 59}{60 - 59} = 3540\]
PART - B

Note
- Write the correct answer in the space provided in the response sheet.
- For each correct response you get 1 mark. For each incorrect response you lose 1/4 mark.

Q.16 Two cogged wheels of which one has 16 cogs and the other 27 cogs, mesh into each other. If the latter turns 80 times in three quarters of a minute, the number of turns made by the other in 8 seconds is _____

Sol.
Number of turns in 45 sec = 80
⇒ 1 sec = \(\frac{80}{45}\)
⇒ 8 sec = \(\frac{80}{45} \times 8\)
Let x = no. of turns made by other in 8 sec.
Now, According to question,
\[16x = 27 \times \frac{80 \times 8}{45}\]
\[x = \frac{27 \times 80 \times 8}{16 \times 45}\]
\[= 24\]

Q.17 If n is a positive integer such that \(a^{2n} = 2\), then \(2a^{6n} - 16\) is _____

Sol.
\[a^{2n} = 2\]
\[2a^{6n} - 16\]
\[= 2(a^{2n})^3 - 2^4\]
\[= 2 \times 2^3 - 2^4\]
\[= 2^4 - 2^4\]
\[= 0\]

Q.18 The least number of children in a family such that every child has at least one sister and one brother is ____

Sol.

```
+-----------------+
| PARENT          |
+-----------------+
|                ↓|
|             SON |
|      SON      |
|     DAUGHTER   |
|      DAUGHTER  |
```
⇒ Least number of children = 4

Q.19 A water tank is \(\frac{4}{5}\) full. When 40 liters of water is removed, it becomes \(\frac{3}{4}\) full. The capacity of the tank in liters is ____

Sol.
Let tank capacity = x litre
Now, According to question
\[\frac{4}{5}x - 40 = \frac{3}{4}x\]
\[\frac{4}{5}x - \frac{3}{4}x = 40\]
\[16x - 15x = 40\]
\[x = 800\]
Q.20  ABC is an equilateral triangle. Squares are described on the sides AB and AC as shown. The value of x is ___

\[ \angle A = \angle 2 = 60^\circ \] [Each Angle of equilateral triangle]

Now, 
\[ \angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ \]
\[ 90^\circ + 60^\circ + 90^\circ + \angle 4 = 360^\circ \]
\[ \angle 4 = 120^\circ \]

\[ \therefore \] ABC is equilateral triangle & squares are described on it sides.

\[ \therefore \] Sides of both squares are also equal

\[ = \angle APQ = \angle AQP \] [Angle opp. to equal sides are equal]

Now, In \( \triangle APQ \),
\[ x + \angle 4 + x = 180^\circ \]
\[ 2x + 120 = 180^\circ \]
\[ \Rightarrow \ x = 30^\circ \]

Q.21  ABCD is a trapezium with AB = 6 cm, AD = 8 cm and CD = 18 cms. The sides AB and CD are parallel and AD is perpendicular to AB. P is the point of intersection of AC and BD. The difference between the areas of the triangles PCD and PAB in square cms is ___
area II = area IV

area of $\triangle ABD = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

area of $\triangle ADC = \frac{1}{2} \times 8 \times 18 = 72 \text{ cm}^2$

Now, $\frac{\text{ar. I}}{\text{ar. II}} = \frac{BP}{PD} \text{ & } \frac{\text{ar. II}}{\text{ar. III}} = \frac{AP}{PC}$

Now, $\triangle APB \sim \triangle CPD$ (By AA similarity rule)

$\Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{AB}{DC} = \frac{6}{18} = \frac{1}{3}$

$\Rightarrow 3AP = PC \text{ & } 3BP = PD$

$\Rightarrow \frac{\text{ar. I}}{\text{ar. II}} = \frac{1}{3}$

$\Rightarrow \frac{\text{ar. II}}{\text{ar. III}} = \frac{1}{3}$

$\therefore I + II = 24$

$I + 3I = 24$

$4I = 24$

$I = 6 \text{ cm}^2$

$II + III = 72$

$\frac{3}{3} + III = 72$

$III = \frac{3 \times 72}{4} = 54 \text{ cm}^2$

$\therefore \text{area III} – \text{area I}$

$= 54 – 6$

$= 48 \text{ cm}^2$

Q.22 The price of cooking oil has increased by 25%. The percentage of reduction that a family should effect in the use of oil so as not to increase the expenditure is ____

Sol. Let price of cooking oil be $x$.

consumption be $y$ lit & let $K\%$ of reduction be there in consumption

Now, ATQ

$(x + 25\% \text{ of } x) (y - K\% \text{ of } y) = xy$
\[
\left( x + \frac{25x}{100} \right) \left( y - \frac{ky}{100} \right) = xy
\]
\[
xy \left( 125 \right) \left( \frac{100 - K}{100} \right) = xy
\]
\[
100 - K = \frac{100 \times 100}{125}
\]
\[
K = 100 - \frac{100 \times 100}{125}
\]
\[
K = \frac{12500 - 10000}{125}
\]
\[
K = \frac{2500}{125} = 20\%
\]

Q.23 The number of natural numbers between 99 and 999 which contains exactly one zero is ____
Sol. Between 100 to 200, we have
101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 120, 130, 140, 150, 160, 170, 180, 190 = 18
Similarly 200 to 300 \(\rightarrow\) 18 & So on
\[\therefore\] Total = 9 \(\times\) 18 = 162

Q.24 In the adjoining figure we have semicircles and AB = BC = CD. The ratio of the unshaded area to the shaded area is ____

Sol. let AB = BC = CD = x

Then total unshaded
Area III + Area I
\[
= \frac{1}{2} \pi \left( \frac{3x}{2} \right)^2 - \frac{1}{2} \pi \left( \frac{2x}{2} \right)^2 + \frac{1}{2} \pi \left( \frac{x}{2} \right)^2
\]
\[
= \frac{1}{2} \pi \left[ \frac{9x^2}{4} - x^2 + \frac{x^2}{4} \right]
\]
\[
= \frac{1}{2} \pi \left[ \frac{9x^2 - 4x^2 + x^2}{4} \right]
\]
\[ = \frac{1}{2} \pi \left( \frac{6x^2}{4} \right) \quad \ldots(1) \]

Area of shaded area = Area II – Area I

\[ = \frac{1}{2} \pi \left( \frac{2x^2}{2} \right) - \frac{1}{2} \pi \left( \frac{x^2}{2} \right) \]

\[ = \frac{1}{2} \pi \left[ x^2 - \frac{x^2}{4} \right] \]

\[ = \frac{1}{2} \pi \times \frac{3x^2}{4} \quad \ldots(2) \]

Eq. (1) ÷ eq. (2) = 2 : 1

Q.25  Gold is 19 times as heavy as water and copper is 9 items as heavy as water. The ratio in which these two metals be mixed so that the mixture is 15 times as heavy as water is _____

Sol. let the mass of 1 volume unit of water be 1 then 1 volume unit of gold has mass 19 and 1 volume unit of copper has mass 9.
Now let g be the volume of gold used and c be that of copper. Then total mass divided by density

\[
\frac{19g + 9c}{g + c} = 15
\]

19g + 9c = 15g + 15c
4g = 6c
\[ \frac{g}{c} = 3 : 2 \]

The mass ratio is 3(19) : 2(9) = 19 : 6

Q.26  Five angles of a heptagon (seven sided polygon) are 160º, 135º, 185º, 145º and 125º. If the other two angles are both equal to xº, then x is ____

Sol. Sum of all the interior angles of heptagon

\[
= (n - 2) \times 180º
\]

\[
= (7 - 2) \times 180º = 5 \times 180º = 900
\]

160º + 135º + 185º + 145º + 125º + x + x = 900

2x = 900 – 750 = 150

x = 75º

Q.27  ABCD is a trapezium with AB parallel to CD and AD perpendicular to AB. If AB = 23 cm, CD = 35 cm and AD = 5 cm. The perimeter of the given trapezium in cms in ___

Sol.
In $\triangle B E C$

$(BC)^2 = (BE)^2 + (EC)^2$

$(BC)^2 = (5)^2 + (12)^2$

$= 25 + 144$

$(BC)^2 = 169$

$BC = 13$ cm

Perimeter of trapezium

$= AB + BC + CD + AD$

$= 23 + 13 + 35 + 5$

$= 76$ cm

Q.28 The number of three digit numbers which are multiples of 11 is __

Sol.

3 digit numbers which are multiple of 11 is

$\Rightarrow 110, 121, 132 \ldots, 990$

in A.P. $= a = 110$

$d = 11$

$a_n = 990$

$a_n = a + (n – 1)d$

$990 = 110 + (n – 1)11$

$\Rightarrow 990 – 110 = (n – 1)11$

$\Rightarrow (n – 1) 11 = 880$

$n – 1 = 80$

$n = 81$

81 terms are there

Q.29 If a, b are digits, ab denotes the number $10a + b$. Similarly, when a, b, c are digits, abc denotes the number $100a + 10b + c$. If $X, Y, Z$ are digits such that $XX + YY + ZZ = XYZ$, then $XX \times YY \times ZZ$ is ___

Sol.

Given :

$XX + YY + ZZ = XYZ$

$\Rightarrow 10X + X + 10Y + Y + 10Z + Z = 100X + 10Y + Z$

$\Rightarrow 11X + 11Y + 11Z = 100X + 10Y + Z$

$\Rightarrow 10Z + Y = 89X$

$X = 1, Y = 9, Z = 8$

So, $XX \times YY \times ZZ = 11 \times 99 \times 88$

$= 95832$

Q.30 The positive integer $n$ has 2, 5 and 6 as its factors and the positive integer $m$ has 4, 8, 12 as its factors. The smallest value of $m + n$ is ___

Sol.

Smallest value of $n$ having 2, 5, 6 as its factor is

$n = 2 \times 3 \times 5$

$n = 30$

smallest value of $m$ having 4, 8, 12 as it factor

$m = 2 \times 2 \times 2 \times 3 = 24$

smallest value of $m + n = 24 + 30 = 54$