KVPY QUESTION PAPER-2017 (STREAM SA)

Date : 05 /11/2017

Section 1-PartA-Mathematics

1. A quadrilateral has distinct integer side lengths. If the second-largest side has length 10, then the maximum possible length of the largest side is **[2017]**

(A) 25 (B) 26 (C) 27 (D) 28

Sol. **[B]**

Let b = 10, c = 9, d = 8

Sum of three sides > IVth side

b + c + d > a

a < 27

2. The largest power of 2 that divides $\frac{200!}{100!}$ is **[2017]**

(A) 98 (B) 99 (C) 100 (D) 101

Sol. **[C]**

Exponent of 2 in 200 ! = 197

Exponent of 2 in 100 ! = 97

3. Let a_1, a_2, a_3, a_4 be real numbers such that $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1$. Then the smallest possible value of the expression $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + (a_4 - a_1)^2$ lies in the interval **[2017]**

(A) (0, 1.5) (B) (1.5, 2.5) (C) (2.5, 3) (D) (3, 3.5)

Sol. **[Bonus]**

$\because\ a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1$

Smallest possible value of $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + (a_4 - a_1)^2 = 0$

if $a_1 = a_2 = a_3 = a_4 = \pm\frac{1}{2}$

(It should be bonus)

4. Let S be the set of all ordered pairs (x, y) of positive integers satisfying the condition $x^2 - y^2 = 12345678$. Then **[2017]**

(A) S is an infinite set (B) S is the empty set

(C) S has exactly one element (D) S is a finite set and has at least two elements.

Sol. **[B]**

$x^2 - y^2 = 12345678$ $(x, y \in I^+)$

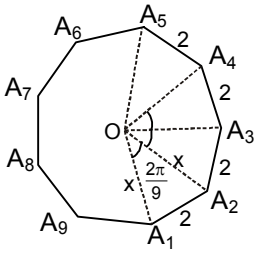
R.H.S. is even, so x, y should be odd integer

but difference of square of two odd integers is multiple of 8 but R.H.S. is not multiple of 8

5. Let $A_1A_2A_3....A_9$ be a nine-sided regular polygon with side length 2 units. The difference between the lengths of the diagonals A_1A_5 and A_2A_4 equals **[2017]**

(A) $2+\sqrt{12}$ (B) $\sqrt{12}-2$ (C) 6 (D) 2

Sol. **[D]**



$\angle A_1OA_2 = \frac{2\pi}{9} = 40°$

$\cos\frac{2\pi}{9} = \frac{x^2+x^2-4}{2x^2}$

$x^2\cos 40°) = x^2 - 2$

$x^2(1-\cos 40°) = 2$

$x = \frac{1}{\sin 20°}$ .....(i)

Now $\cos\frac{8\pi}{9} = \frac{x^2+x^2-(A_1A_5)^2}{2x^2}$ (in ΔA_1OA_5)

$(A_1A_5)^2 = 2x^2\left(1-\cos\frac{8\pi}{9}\right)$

$= 2x^2(1-\cos 160°)$

$= 4x^2\sin^2 80°$

$A_1A_5 = 2x\sin 80°$ .....(ii)

Similarly in ΔA_2OA_4

$A_2A_4 = 2x\sin 40°$ .....(iii)

(ii) – (iii)

$A_1A_5 - A_2A_4 = 2x(\sin 80° - \sin 40°)$

$= 2$ (using (i))

6. Let $a_1, a_2,....a_n$ be n nonzero real numbers, of which p are positive and remaining are negative. The number of ordered pairs (j, k), j < k, for which a_ja_k is positive, is 55. Similarly, the number of ordered pairs (j, k), j < k, for which a_ja_k is negative is 50. Then the value of $p^2 + (n-p)^2$ is **[2017]**

(A) 629 (B) 325 (C) 125 (D) 221

Sol. **[C]**

${}^pC_2 + {}^{n-p}C_2 = 55$

$\frac{p(p-1)}{2} + \frac{(n-p)(n-p-1)}{2} = 55$ ....(i)

Also, $p(n-p) = 50$ ....(ii)

Put in (i)

$$p(p-1)+\frac{50}{p}\left(\frac{50}{p}-1\right)=110$$

$$p^2-p+\left(\frac{50}{p}\right)^2-\frac{50}{p}=110$$

$$p^2-p+\left(\frac{50}{p}\right)^2-\frac{50}{p}=110$$

$$\left(p+\frac{50}{p}\right)^2-100-\left(p+\frac{50}{p}\right)=110$$

$t^2 - t - 210 = 0$

$t = 15$ or -14 (not true)

$$\therefore\ p+\frac{50}{p}=15$$

$\therefore$ To find $p^2 + (n-p)^2 = p^2+\left(\frac{50}{p}\right)^2$

$$=\left(p+\frac{50}{p}\right)^2-100$$

$= 125$ (using (iii))

7. If a, b, c, d are four distinct numbers chosen from the set {1,2,3 ....,9}, then the minimum value of $\frac{a}{b}+\frac{c}{d}$ is **[2017]**

(A) $\frac{3}{8}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{25}{72}$

Sol. **[D]**

$$\frac{2}{9}+\frac{1}{8}=\frac{25}{72}$$

8. If $72^x \cdot 48^y = 6^{xy}$, where x and y are nonzero rational numbers, then x + y equals **[2017]**

(A) 3 (B) $\frac{10}{3}$ (C) -3 (D) $-\frac{10}{3}$

Sol. **[D]**

$$72^x \cdot 48^y = 6^{xy}$$

$$3^{2x+y} \cdot 2^{3x+4y} = 2^{xy} \cdot 3^{xy}$$

compare $2x+y = xy$ .....(i)

& $3x + 4y = xy$ .....(ii)

From (i) & (ii) $x = -3y$

put in (i) $-5y = -3y^2$

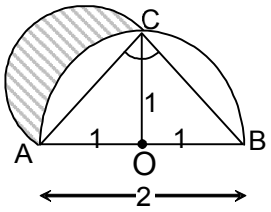
$\Rightarrow y = \frac{5}{3}$

so $x + y = -2y = -\frac{10}{3}$

9. Let AB be a line segment of length 2. Construct a semicircle S with AB as diameter. Let C be the midpoint of the arc AB. Construct another semicircle T external to the triangle ABC with chord AC as diameter. The area of the region inside the semicircle T but outside S is **[2017]**

(A) $\frac{\pi}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}}$

Sol. **[B]**



$\angle ACB = 90°$

AC = $\sqrt{2}$

Required area = Area of semicircle having AC as diameter – Area under arc OAC but outside triangle AOC

$= \frac{1}{2}\left(\pi\left(\frac{\sqrt{2}}{2}\right)^2\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{4} - \frac{\pi}{4} + \frac{1}{2} = \frac{1}{2}$

10. Let r(x) be the remainder when the polynomial $x^{135} + x^{125} - x^{115} + x^5 + 1$ is divided by $x^3 - x$. Then **[2017]**

(A) r(x) is the zero polynomial (B) r(x) is a nonzero constant

(C) degree of r(x) is one (D) degree of r(x) is two

Sol. **[C]**

$x^{135} + x^{125} - x^{115} + x^5 + 1 = k(x^3 - x) + Ax^2 + Bx + C$

put x = 0 $\boxed{C = 1}$

Put x = 1

3 = A + B + 1

$A + B = 2$ .....(i)

put $x = -1$

$-1 = A - B + 1$

$A - B = -2$ .....(ii)

(i) + (ii)

$A = 0$

$B = 2$

11. It is given that the number 43361 can be written as a product of two distinct prime numbers p_1, p_2. Further, assume that there are 42900 numbers which are less than 43361 and are co-prime to it. Then, $p_1 + p_2$ is **[2017]**

(A) 462 (B) 464 (C) 400 (D) 402

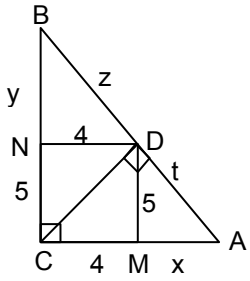
Sol. **[A]**

$43361 = 131 \times 331$

12. Let ABC be a triangle with $\angle C = 90°$. Draw CD perpendicular to AB. Choose points M and N on sides AC and BC respectively such that DM is parallel to BC and DN is parallel to AC. If DM = 5, DN = 4, then AC and BC are respectively equal to **[2017]**

(A) $\frac{41}{4}, \frac{41}{5}$ (B) $\frac{39}{4}, \frac{39}{5}$ (C) $\frac{38}{4}, \frac{38}{5}$ (D) $\frac{37}{4}, \frac{37}{5}$

Sol. **[A]**



$CD = \sqrt{41}$

$\Delta AMD \sim \Delta DNB$

$\frac{y}{5} = \frac{4}{x} = \frac{z}{t}$

$\boxed{xy = 20}$ .....(i)

In ΔBCD

$(y + 5)^2 = 41 + z^2$ .....(ii)

In ΔADC

$(x + 4)^2 = 41 + t^2$ .....(iii)

$$\Rightarrow \frac{(y+5)^2 - 41}{(x+4)^2 - 41} = \left(\frac{z}{t}\right)^2$$

$$\Rightarrow \frac{\left(\frac{20}{x} + 5\right)^2 - 41}{(x+4)^2 - 41} = \left(\frac{4}{x}\right)^2 \text{ (use (i))}$$

$$\Rightarrow \frac{(20+5x)^2-41x^2}{(x+4)^2-41}=16$$

$$\Rightarrow \frac{400+25x^2+200x-41x^2}{x^2+16+8x-41}=16$$

$\Rightarrow -16x^2 + 200x + 400 = 16x^2 + 128x - 400$

$\Rightarrow 32x^2 - 72x - 800 = 0$

$\Rightarrow 4x^2 - 9x - 100 = 0$

$$\boxed{x=\frac{25}{4}}$$

so $y=\frac{20\times 4}{25}=\frac{16}{5}$

13. Let A, G and H be the arithmetic mean, geometric mean and harmonic mean, respetively of two distinct positive real numbers. If α is the smallest of the two roots of the equation $A(G-H)x^2 + G(H-A)x + H(A-G) = 0$, then **[2017]**

(A) $-2 < \alpha < -1$ (B) $0 < \alpha < 1$ (C) $-1 < \alpha < 0$ (D) $1 < \alpha < 2$

Sol. **[B]**

$x = 1$ is a root as sum of coefficient = 0

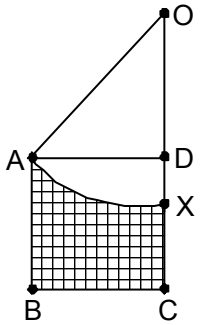
Now $\alpha\beta=\frac{H(A-G)}{A(G+H)}$

Put $\beta = 1$

$$\alpha=\frac{HA-HG}{AG-AH}=\frac{G^2-HG}{AG-AH}$$

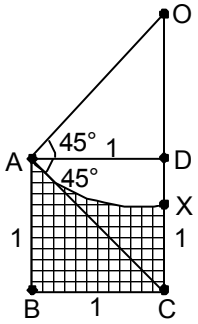
$$=\frac{G(G-H)}{A(G-H)}=\frac{G}{A}<1 \quad \text{[as A.M. > G.M.]}$$

14. In the figure, ABCD is a unit square. A circle is drawn with centre O on the extended line CD and passing through A. If the diagonal AC is tangent to the circle, then the area of the shaded region is **[2017]**



(A) $\frac{9-\pi}{6}$ (B) $\frac{8-\pi}{6}$ (C) $\frac{7-\pi}{4}$ (D) $\frac{6-\pi}{4}$

Sol. **[D]**



$\therefore \angle OAC = 90°$ as AC is tangent and OA is radius

as $\angle CAD = 45°$

So $\angle OAD = 45° = \angle AOD$

$\therefore OA = \sqrt{2}$

Area of shaded region

= 1 – (area of sector OAX – area of ΔOAD)

$$= 1-\left[\frac{1}{2}\times 2\times\frac{\pi}{4}-\frac{1}{2}\right]$$

$$= 1-\left[\frac{\pi}{4}-\frac{1}{2}\right]=\frac{3}{2}-\frac{\pi}{4}$$

15. The sum of all non-integer roots of the equation $x^5 - 6x^4 + 11x^3 - 5x^2 - 3x + 2 = 0$ is **[2017]**

(A) 6 (B) –11 (C) –5 (D) 3

Sol. **[D]**

$(x - 1)(x - 2)(x^3 - 3x + 1) = 0$

$\therefore$ Required sum of roots = 3

Section 2 - Part-Physics

16. Consider the following statements (X and Y stand for two different elements) **[2017]**

(I) ${}_{32}X^{65}$ and ${}_{33}Y^{65}$ are isotopes.

(II) ${}_{42}X^{86}$ and ${}_{42}Y^{85}$ are isotopes.

(III) ${}_{85}X^{174}$ and ${}_{88}Y^{177}$ have the same number of neutrons.

(IV) ${}_{92}X^{235}$ and ${}_{94}Y^{235}$ are isobars.

The correct statements are :

(A) II and IV only. (B) I, II and IV only. (C) II, III and IV only. (D) I, II, III and IV only.

Sol. **[C]**

Isotopes : molecules haveing same number of proton

Isobars : Molecules haveing same number of nucleons -

17. A student performs an experiment to determine the acceleration due to gravity g. The student throws a steel ball up with initial velocity u and measures the height h travelled by it at different times t. The graph the student should plot on a graph paper to readily obtain the value of g is **[2017]**

(A) h versus t. (B) h versus t^2 (C) h versus $\sqrt{t}$. (D) $\frac{h}{t}$ versus t.

Sol. **[D]**

$h = ut + \frac{1}{2} at^2$

$h = ut - \frac{1}{2} gt^2$

$\frac{h}{t} = u - \frac{1}{2} gt$

$y = mx + c$

stope will define the value gravity.

18. A person goes from point P to point Q covergin 1/3 of the distance with speed 10km/hr, the next 1/3 of the distance at 20 km/hr and the last 1/3 of the distance at 60 km/hr. The average speed of the person is **[2017]**

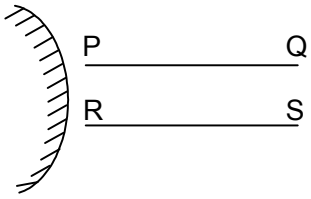
(A) 30 km/hr (B) 24 km/hr (C) 18 km/hr (D) 12 km/hr

Sol. **[C]**

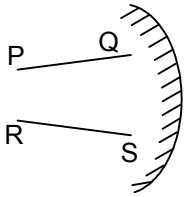
$$V_{avg} = \frac{\text{Total Distance}}{\text{total time}} = \frac{S}{\frac{S/3}{10} + \frac{S/3}{20} + \frac{S/3}{60}}$$

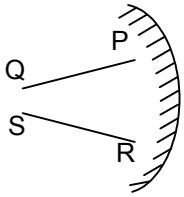
$V_{avg} = 18$ km/hr

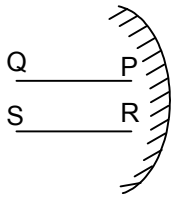
19. A person looks at the image of two parallel finite length lines PQ and RS in a convex mirror (see figure).

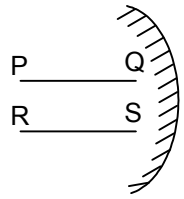


Which of the following represents schematically the image correctly ? (Note : Letters P, Q, R and S are used only to denote the endpoints of the lines.) **[2017]**

A.


B.


C


D.


(A) A (B) B (C) C (D) D

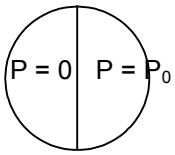
Sol. **[B]**

Object placed in front of mirror. For all position of object infornt of mirror, image in virtural, erect, smaller in size. As object is moved away from pole magnification decrases.

20. In Guericke's experiment to show the effect of atmospheric pressure, two copper hemispheres were tightly fitted to each other to form a hollow sphere and the air from the sphere was pumped out to create vacuum inside. If the radius of each hemisphere is R and the atmospheric pressure is P, then the minimum force required (when the two hemispheres are pulled apart by the same force) to separate the hemispheres is **[2017]**

(A) $2\pi R^2P$ (B) $4\pi R^2P$ (C) πR^2P (D) $\pi R^2P/2$

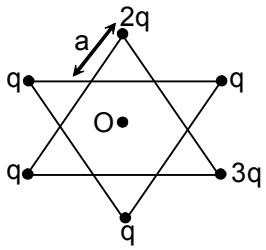
Sol. **[C]**



$F = (P_0 - 0)A$

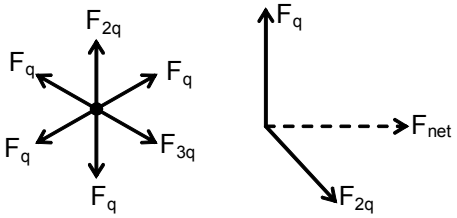
$F = \pi R^2P$

21. Positive point charges are placed at the vertices of a star shape as shown in the figure. Direction of the electrostatic force on a negative point charge at the centre O of the star is **[2017]**

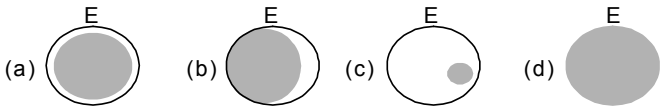


(A) towards right (B) vertically up (C) towards left (D) vertically down

Sol. **[A]**



22. A total solar eclipse is observed from the earth. At the same an observer on the moon views the earth. She is most likely to see (E denotes the earth) **[2017]**



(A) a (B) b (C) c (D) d

Sol. **[B]**

Theoritical

23. Ice in a freezer is at –7°C. 100 g of this ice is mixed with 200 g of water at 15°C. Take the freezing temperature of water to be 0°C, the specific heat of ice equal to 2.2 J/g °C, specific heat of water equal to 4.2 J/g °C, and the latent heat of ice equal to 335 J/g. Assuming no loss of heat to the environment, the mass of ice in the final mixture is closest to **[2017]**

(A) 88 g (B) 67 g (C) 54 g (D) 45 g

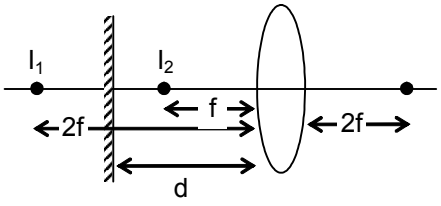
Sol. **[B]**

Let mgm of ice melted
Mixture will be at 0ºC
Heat given = 200 × (4.2) (15) J
Heat absorbed = 100 × 2.2 × (7) + m(335)
by solving m = 33 gm
So ice remain = 100 – 33 = 67g

24. A point source of light is placed at 2f from a converging lens of focal length f. A flat mirror is placed on the other side of the lens at a distanc d such that rays reflected from the mirror are parallel after passing through the lens again. If f = 30 cm, then d is equal to **[2017]**

(A) 15 cm. (B) 30 cm. (C) 45 cm. (D) 75 cm.

Sol. **[C]**



at 2f – object then image will also be at '2f'. Finally image is at ∞ therefore after reflection from mirror. Image must be formed at focus.

$$d = \frac{f + 2f}{2} = 45 \text{ cm}$$

25. The word ''KVPY'' is written on a board and viewed through different lense such that board is at a distance beyond the focal length of the lens. **[2017]**

(i) (ii)

Ignorging magnification effects, consider the following statements

(I) Image (i) has been viewed from the planar side of a plano-convex lens and image (ii) from the convex side of a plano-convex lens.

(II) Image (i) has been viewed from the concave side of a plano-concave lens and image (ii) from the planar side of a plano-convex lens.

(iii) Image (i) has been viewed from the cocave side of a plano-concave lens and image (ii) from the planar side of a plano-convex lens.

(iv) Image (i) has been viewed from the planar side of a plano-concave lens and image (ii) from the convex side of a plano-convex lens.

Which of the above statements are correct ?

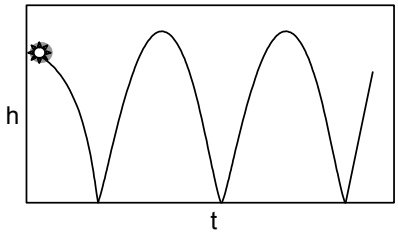
(A) All four. (B) Only (III). (C) Only (IV). (D) Only (II), (III) and (IV).

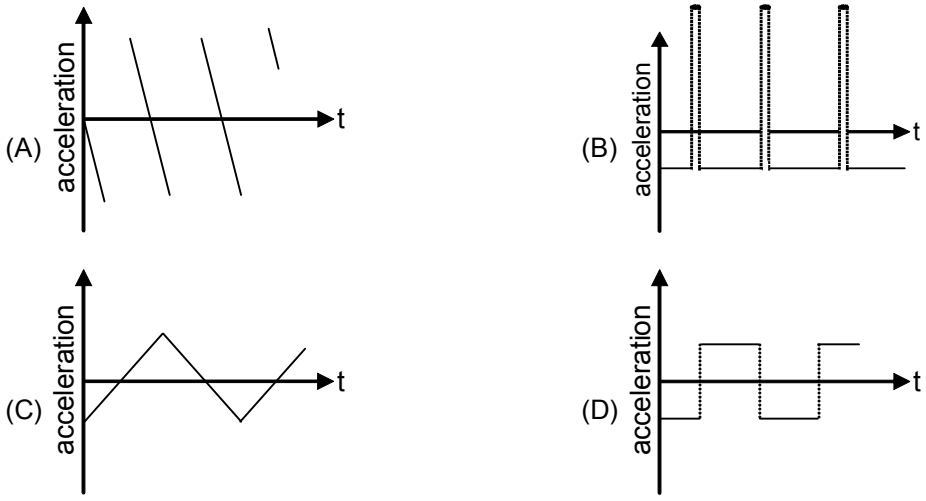
Sol. **[D]**

(i) For plano-concave lens or concave lens if object is placed beyond focus image is erected

(ii) For convex lens If object is placed beyond focus image is inverted

26. A ball is dropped vertically from heigth h and is bouncing elastically on the floor (see figure). Which of the following plots best depicts the acceleration of the ball as a function of time. **[2017]**





Sol. **[B]**
Acceleration is all the time (–g) except at the time of collision it is impulsive force in (+)ve decrease.

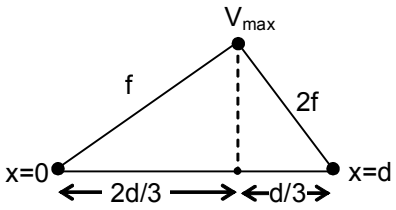
27. A student studying the similarities and differences between a camera and the human eye makes the following observations. **[2017]**
(I) Both the eye and the camera have convex lenses.
(II) In order to focus, the eye lens expands or contracts while the camera lens moves forward or backward.
(III) The camera lens produces upside down real images while the eye lens produces only upright real image.
(IV) A screen in camera is equivalent to the retina in the eyes.
(V) A camera ajusts the amount of light entering in it by adjusting the apeture of the lens. In the eye the cornea controls the amount of light.
The correct statemetns are :
(A) Only (I), (II) (IV). (B) Only (I), (III), (V).
(C) Only (I), (II), (IV), (V). (D) All

Sol. **[A]**
Theoritical

28. A particle starts moving along a line from zero initial velocity and comes to rest after moving distance d. During its motion it had a constant acceleration f over 2/3 of the distance, and covered the rest of the distance with constant retardation. The time taken to cover the distance is **[2017]**

(A) $\sqrt{2d/3f}$ (B) $2\sqrt{d/3f}$ (C) $\sqrt{3d/f}$ (D) $\sqrt{3d/2f}$

Sol. **[C]**



$a_1 l_1 = a_2 l_2$
by equation of motion
Retardation = 2f
$t = t_1 + t_2$

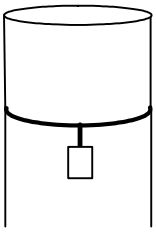
$$= \sqrt{\frac{4d}{3f}} + \sqrt{\frac{2d}{2f}} = \sqrt{\frac{3d}{f}}$$

29. If the image formed by a thin convex lens of power P has agnification m then image distance ν is **[2017]**

(A) $\nu = \frac{1-m}{P}$ (B) $\nu = \frac{1+m}{P}$ (C) $\nu = \frac{m}{P}$ (D) $\nu = \frac{1+2m}{P}$

Sol. **[A]**
Theoritical (formula)

30. A long cylindrical pipe of radius 20 cm is closed at its upper end and has an airtight piston of negligible mass as shown. When a 50 Kg mass is attached to the other end of the pistion, it moves down. If the air in the enclosure is cooled from temperature T to $T - \Delta T$, the piston moves back to its original position. Then $\Delta T/T$ is close to (Assuming air to be an ideal gas, g = 10 m/s^2, atmospheric pressure is 10^5 Pascal), **[2017]**



(A) 0.01 (B) 0.02 (C) 0.04 (D) 0.09

Sol. **[C]**

Initially pressure = P_0

When 50 kg mass is suspended

$$\text{pressure} = P_0 - \frac{mg}{A}$$

(temp = constant)

$$P_0V_i = \left(V_0 - \frac{mg}{A}\right)V_f$$

$$\left(P_0 - \frac{mg}{A}\right)V_f = nRT$$

$$\left(P_0 - \frac{mg}{A}\right)V_i = nR(T - \Delta T)$$

$$\frac{V_f}{V_i} = \frac{T}{T - \Delta T}$$

$$\frac{T - \Delta T}{T} = \frac{V_i}{V_f}$$

$$1 - \frac{\Delta T}{T} = \left(P_0 - \frac{mg}{A}\right)\frac{1}{P_0}$$

$$\frac{\Delta T}{T} = \frac{mg}{AP_0}$$

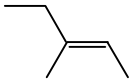
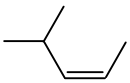
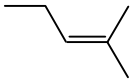
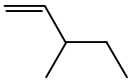
$$= \frac{50 \times 10}{3.14 \times (0.2)^2 \times 10^5}$$

$$= \frac{500}{12.56 \times 10^3}$$

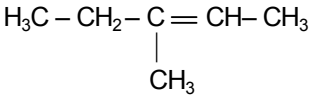
$$= \frac{5}{125.6} = \frac{1}{25} = 0.04$$

Section 3-Part A-Chemistry

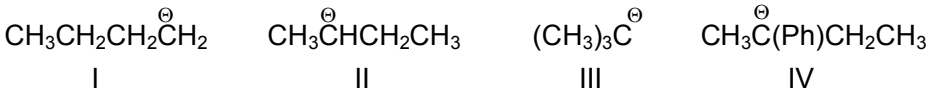
31. The structure of 3-methylpent-2-ene is **[2017]**

(A)  (B)  (C)  (D) 

Sol. **[A]**



32. The stability of carbanions **[2017]**



I II III IV

follows the order

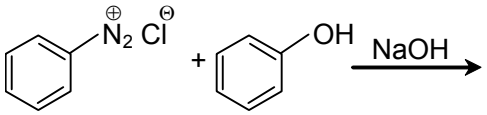
(A) III < IV < I < II (B) I < II < IV < III (C) III < II < I < IV (D) IV < III < II <I

Sol. **[C]**

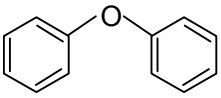
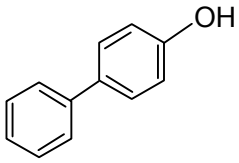
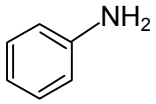
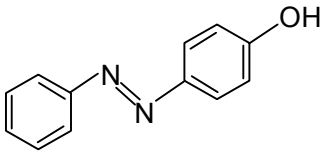
(IV) is stabilized by Resonance &

1° > 2° > 3° $\rightarrow$ stability order of carbanion

33. In the following reaction

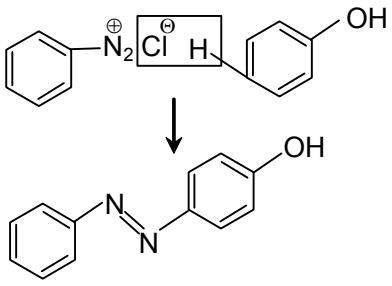


the major product is **[2017]**

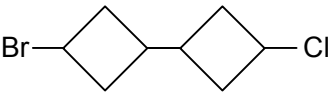
(A)  (B)  (C)  (D) 

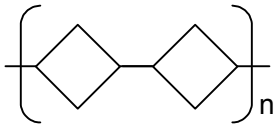
Sol. **[D]**

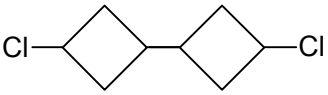
Coupling reaction



34. In the reaction of 1-bromo-3-Chlorocyclobutane with two equivalents of sodium in ether, the major product is **[2017]**

(A) 

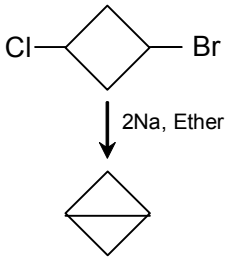
(B) 

(C) 

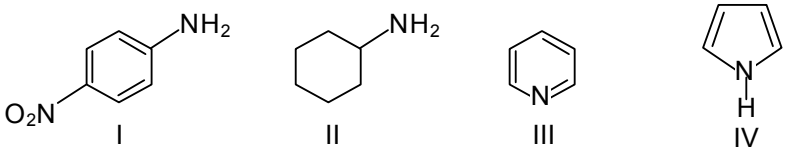
(D) 

Sol. **[D]**

Wurtz Reaction



35. The order of basicity of



in water is **[2017]**

(A) IV < III < I < II (B) II < I < IV < III (C) IV < I < III < II (D) II < III < I < IV

Sol. **[C]**

(II) is most basic because its lone pair is not participating in resonance & attacted to sp^3 hyb. carbon.

36. The first ionisation energy of Na, B, N and O atoms follows the order **[2017]**

(A) B < Na < O < N (B) Na < B < O < N (C) Na < O < B < N (D) O < Na < N < B

Sol. **[B]**

N has half filled stable configuration and Na is most electropositive element.

37. Among P_2O_5, As_2O_3, Sb_2O_3 and Bi_2O_3 the most acidic oxide is **[2017]**

(A) P_2O_5 (B) As_2O_3 (C) Sb_2O_3 (D) Bi_2O_3

Sol. **[A]**

Top to down, Basic strength increases.

38. Among K, Mg, Au and Cu, the one which is extracted by heating its ore in air is **[2017]**

(A) K (B) Mg (C) Au (D) Cu

Sol. **[D]**

Theoritical question.

39. The metal ion with total number of electrons same as S^{2-} is **[2017]**

(A) N^{a+} (B) Ca^{2+} (C) Mg^{2+} (D) Sr^{2+}

Sol. **[B]**

S^{-2} has 18 electrons & so Ca^{+2} ion are having

40. X g of Ca [atomic mass = 40] dissolves completely in concentrated HCl solution to produce 5.04 L of H_2 gas at STP. The value of X is closest to **[2017]**

(A) 4.5 (B) 8.1 (C) 9.0 (D) 16.2

Sol. **[C]**

$Ca + 2HCl \rightarrow CaCl_2 + H_2(g)$

$\frac{x}{40}$ $\frac{5.04}{22.4}$

$\Rightarrow \frac{x}{40} = \frac{5.04}{22.4} \Rightarrow \boxed{x = 9}$

41. A 20 g object is moving with velocity 100 ms^{-1}. The de Broglie wavelength (in m) of the object is [Planck's constant h = 6.626 $\times 10^{-34}$ J s] **[2017]**

(A) 3.313×10^{-34} (B) 6.626×10^{-34}

(C) 3.313×10^{-31} (D) 6.626×10^{-31}

Sol. **[A]**

$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-31}}{20 \times 10^{-3} \times 100} = 3.313 \times 10^{-34}$

42. In a closed vessel at STP, 50 L of CH_4 is ignited with 750 L of air (containing 20% O_2). The number of moles of O_2 remaining in the vessel on cooling to room temperature is closest to **[2017]**

(A) 5.8 (B) 2.2 (C) 4.5 (D) 6.7

Sol. **[B]**

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

50 L 150 L

CH_4 is limiting reagent

Moles of O_2 remaining = $\frac{50}{22.4}$ = 2.2 moles

43. CO_2 is passed through lime water. Initially the solution turns milky and then becomes clear upon continued boubbling of CO_2. The clear solution is due to the formation of **[2017]**

(A) $CaCO_3$ (B) CaO (C) $Ca(OH)_2$ (D) $Ca(HCO_3)_2$

Sol. **[D]**

$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 + H_2O$

$CaCO_3 + H_2O \xrightarrow{CO_2} Ca(HCO_3)_2$

Clear solution

44. The maximum number of electrons that can be filled in the shell with the principal quantum number n = 3 is **[2017]**

(A) 18 (B) 9 (C) 8 (D) 2

Sol. **[A]**
3s, 3p & 3d contains total 18 electrons.

45. The atomic radii of Li, F, Na and Si follow the order. **[2017]**
(A) Si > Li > Na > F (B) Li > F > Si > Na (C) Na > Si > F > Li (D) Na > Li > Si > F

Sol. **[D]**
Factual

Section 4 - Part A - Biology

46. The major excretory product of birds is **[2017]**
(A) urea (B) uric acid (C) nitrates (D) ammonia

Sol. **[B]**
For water conservation

47. Codon degeneracy means that **[2017]**
(A) several of the amino acids are coded by more than one codon
(B) one codon can code for many amino acids
(C) one amino acid can be coded by only one codon
(D) The codons are triplet nucleotide sequences

Sol. **[A]**
Wobbling phenomenon degreneracy occurs at 3rd base of codon

48. In cell cyle, during interphase, **[2017]**
(A) two daughter cells are produced (B) the nucleus is divided into two daughter nuclei
(C) the chromosome condenses (D) the DNA is replicated

Sol. **[D]**
DNA replicates in S-phase of interphase

49. Transfer of genetic material between populations is best defined as **[2017]**
(A) gene flow (B) genetic drift (C) genetic shift (D) speciation

Sol. **[A]**
Definition of gene flow - transfer of gene or allele from one population to another

50. Which ONE of the following statements is CORRECT about the tobacco mosaic virus ? **[2017]**
(A) It affects all monocotyledonous plants
(B) It affects photosynthetic tissue of the infected plant
(C) It does not infect other species beloging to the Solanaceae
(D) It infects gymnosperms

Sol. **[B]**
Fact based

51. Which ONE of the following statements is CORRECT about placenta ? **[2017]**
(A) Placenta is permeable to all bacteria
(B) Oxygen and carbon dioxide cannot diffuse through the placenta
(C) Waste products diffuse out of placenta into maternal blood
(D) Placenta does not secrete chorionic gonadotropins

Sol. **[C]**
Because it is permeable for ammonia and other Nitrogenous wastes.

52. The respiratory quotient of the reaction given below is **[2017]**

$2(C_{51}H_{98}O_6) + 145\ O_2 \longrightarrow 102\ CO_2 + 90\ H_2O + \text{energy}$

(A) 0.703 (B) 0.725 (C) 0.960 (D) 1.422

Sol. **[A]**

$$RQ = \frac{CO_2}{O_2} = \frac{102}{145} = 0.703$$

53. Which ONE of the following statements is INCORRECT about nucleosomes ? **[2017]**

(A) They contain DNA (B) They contain histones

(C) They are membrane-bound organelle (D) They are a part of chromosomes

Sol. **[C]**

Nucleosome - DNA wrapped around histone during DNA packaging

54. The immediate precursor of thyroxine is **[2017]**

(A) tyrosine (B) tryptophan (C) pyridoxine (D) thymidine

Sol. **[A]**

Tyrosine with Iodine form thyroxine.

55. The maximum number of oxygen molecules that can bind to one molecule of hemoglobin is **[2017]**

(A) 8 (B) 6 (C) 4 (D) 2

Sol. **[C]**

Due to presence of four hememolecule.

56. Which ONE of the following biomolecules is synthesized in smooth endoplasmic reticulum ? **[2017]**

(A) Proteins (B) Lipids (C) Carbohydrates (D) Nucleotides

Sol. **[B]**

SER functions for lipid synthesis

57. The products of light reaction during photosynthesis include **[2017]**

(A) ATP and NADPH (B) O_2 and $NADP^+$ (C) O_2 and H_2O (D) $NADP^+$ and H_2O

Sol. **[A]**

ATP & NADPH is assimilatory power produced in light reaction i.e. used in dark reaction.

58. Hypothalamus directly controls the production of which of the following hormones ? **[2017]**

(A) glucocorticoid and insulin (B) insulin and glucagon

(C) atrial natriuretic factor and gastrin (D) glucocorticoids and androgens

Sol. **[D]**

By ACTH-RH which act on adenohypophysis for release of ACTH, ACTH than stimulate adrenal cortex for production of glucocorticoids and androgens

59. Which ONE of the following drugs is NOT obtained from fungal or plant sources ? **[2017]**

(A) Penicillin (B) Reserpine (C) Acetaminophen (D) Quinine

Sol. **[C]**

Acetaminophen is paracetamol not produced by plant or fungi, it is artificially formed

60. Jean-Baptiste Lamarck explained evolution based on **[2017]**

(A) natural selection (B) survival of the fittest

(C) mutations (D) inheritance of acquired characteristics

Sol. **[D]**

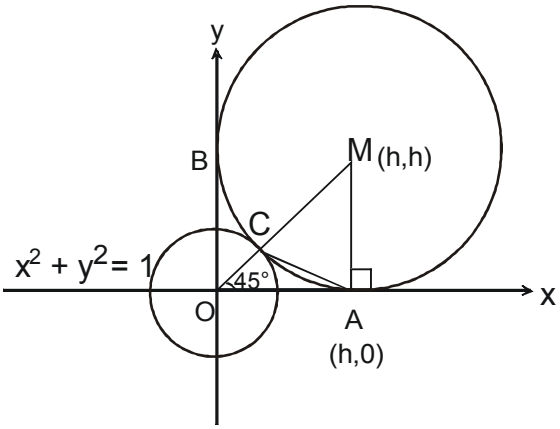
Inheritance of acquired character.

Section 5- Part B- Mathematics

61. Let S be the circle in xy-plane which touches the x-axis at point A, the y-axis at point B and the unit circle $x^2 + y^2 = 1$ at point C externally. If O denotes the origin, then the angle OCA equals **[2017]**

(A) $\frac{5\pi}{8}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{3\pi}{5}$

Sol. **[A]**



$\therefore$ Slope of OM = 1

$\therefore \angle COA = 45°$

$C\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Now OM = OC + CM

$\sqrt{2}h = 1 + h$

Squaring

$2h^2 = h^2 + 1 + 2h$

$h^2 - 2h - 1 = 0$

$h = \frac{2 \pm \sqrt{8}}{2} \Rightarrow h = 1 \pm \sqrt{2}$

$A(1+\sqrt{2}, 0)$

Slope of AC = $\frac{0 - \frac{1}{\sqrt{2}}}{1 + \sqrt{2} - \frac{1}{\sqrt{2}}} = -\frac{1}{\sqrt{2}+1} = -(\sqrt{2}-1)$

$= -\tan 22\frac{1}{2}^\circ = \tan(180 - 22\frac{1}{2})^\circ = \tan 157\frac{1}{2}^\circ$

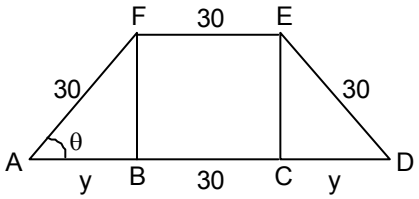
$\therefore \angle CAX = 157\frac{1}{2}^\circ$

$\therefore \angle OCA = 157\frac{1}{2}^\circ - 45^\circ = 112\frac{1}{2}^\circ = \frac{5\pi}{8}$

62. In an isosceles trapezium, the length of one of the parallel sides, and the lengths of the non-parallel sides are all equal to 30. In order to maximize the area of the trapezium, the smallest angle should be **[2017]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Sol. **[C]**



In ΔABF, $y = 30\cos\theta$

Area is $A = \frac{1}{2}[60 + 60\cos\theta](30\sin\theta)$

$= 30\,(1+\cos\theta)\,(30\sin\theta)$

$= 900\,(\sin\theta + \sin\theta\cos\theta)$

For maximum or minimum

$\frac{dA}{d\theta} = 900[\cos\theta + (-\sin^2\theta + \cos^2\theta)] = 0$

$\cos\theta - 1 + \cos^2\theta + \cos^2\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$ (not possible)

$\theta = 60^\circ$

63. Let A_1, A_2, A_3 be regions in the xy-plane defined by

$A_1 = \{(x, y) : x^2 + 2y^2 \le 1\}$,

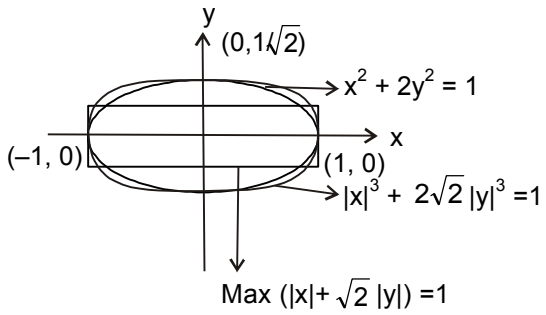
$A_2 = \{(x, y) : |x|^3 + 2\sqrt{2}\,|y|^3 \le 1\}$,

$A_3 = \{(x, y) : \max(|x|, \sqrt{2}\,|y|) \le 1$. Then **[2017]**

(A) $A_1 \supset A_2 \supset A_3$ (B) $A_3 \supset A_1 \supset A_2$ (C) $A_2 \supset A_3 \supset A_2$ (D) $A_3 \supset A_2 \supset A_1$

Sol. **[D]**

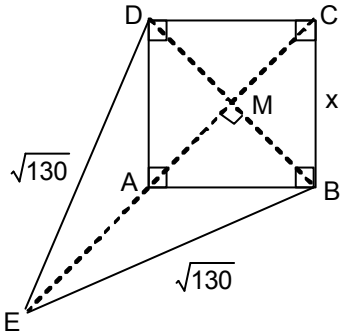
Draw Figure



64. Let ABCD be a square and E be a point outside ABCD such that E, A, C are collinear in that order. Suppose EB = ED = $\sqrt{130}$ and the areas of triangle EAB and square ABCD are equal. Then the area of square ABCD is **[2017]**

(A) 8 (B) 10 (C) $\sqrt{120}$ (D) $\sqrt{125}$

Sol. **[B]**



given : ar ΔEAB = ar square ABCD

EB = ED = $\sqrt{130}$

Let side of square = x

$$BM = \frac{x}{\sqrt{2}} = AM$$

arΔEAB = arΔEMB – arΔABM

$$= \frac{1}{2}EM \times BM - \frac{1}{2}AM \cdot BM$$

$$= \frac{1}{2}\left(\sqrt{130 - \frac{x^2}{2}}\right)\frac{x}{\sqrt{2}} - \frac{1}{2}\left(\frac{x}{\sqrt{2}}\right)^2$$

Area of square = x^2

Using (i)

$$\frac{1}{2}\frac{x}{\sqrt{2}}\sqrt{130 - \frac{x^2}{2}} - \frac{1}{2}\frac{x^2}{2} = x^2$$

$$\frac{x}{2\sqrt{2}}\sqrt{130-\frac{x^2}{2}}=\frac{5x^2}{4}$$

solve $x=\sqrt{10}$

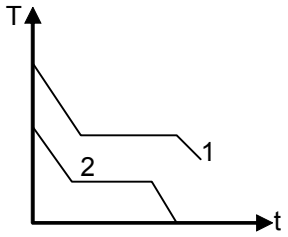
$\therefore\ x^2 = 10$

65. Consider the set A = {1, 2, 3, ..., 30}. The number of ways in which one can choose three distinct numbers from A so that the product of the chosen numbers is divisible by 9 is **[2017]**

(A) 1590 (B) 1505 (C) 1110 (D) 1025

Section 6 - Part B-Physics

66. Two different liquids of same mass are kept in two identical vessels, which are placed in a freezer that extracts heat from them at the same rate causing each liquid to transform into a solid. The schematic figure below shows the temperature T vs time t plot for the two materials. We denote the specific heat in the liquid status to be C_{L1} and C_{L2} for materials 1 and 2 respectively, and latent heats of fusion U_1 and U_2 respectively. **[2017]**



Choose the correct option.

(A) $C_{L1} > C_{L2}$ and $U_1 < U_2$ (B) $C_{L1} > C_{L2}$ and $U_1 > U_2$

(C) $C_{L1} < C_{L2}$ and $U_1 > U_2$ (D) $C_{L1} < C_{L2}$ and $U_1 < U_2$

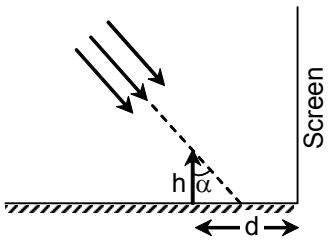
Sol. **[C]**

Let Refrigerater extract Q joul/per second

$Q.t \Rightarrow ms\,(T_f - T)$

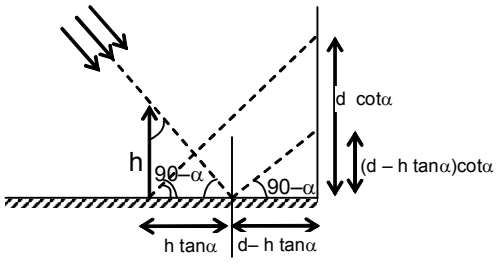
Higher the specific heat, Higher the slope

67. A long horizontal mirror is next to a vertical screen (See figure). Parallel light rays are falling on the mirror at an angle α from the vertical. If a vertical object of height *h* is kept on the mirror at a distance d > h tan (α). The length of the shadow of the object on the screen would be **[2017]**

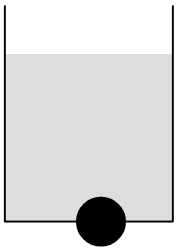


(A) h/2 (B) h tan (α) (C) 2h (D) 4h

Sol. **[C]**



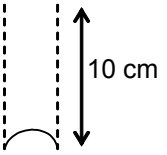
68. A spherical marble of radius 1 cm is stuck in a circular hole of radius slightly smaller than its own radius (for calculation purpose, both can be taken same) at the bottom of a bucket of height 40 cm and filled with water up to 10 cm. If the mass of the marble is 20 g, the net force on the marble due to water is close to **[2017]**



(A) 0.02 N upward (B) 0.02 N downward (C) 0.04 N upward (D) 0.04 N downward

Sol. **[D]**

F_{net} = weight of water over it

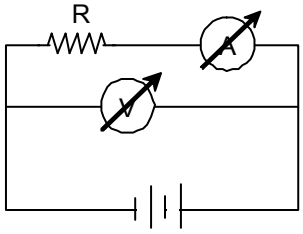


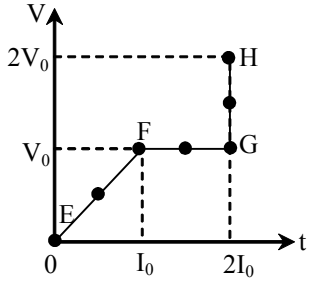
$= (\rho gh)A - \left(\frac{2}{3}\pi R^3\right)\rho g$

$= A(10^3 \times 10 \times 10 \times 10^{-2}) - \frac{2}{3}\pi \times 10^{-2} \times 10^3 \times 10$

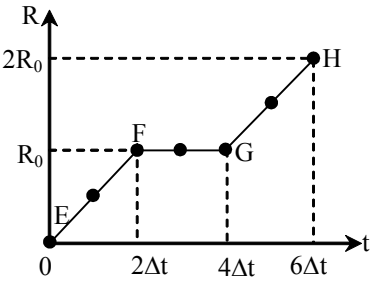
Area = 0.04 N dowanward

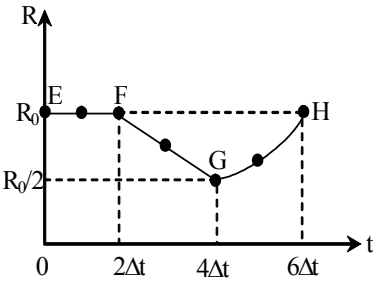
69. In the circuit shown below (on the left) the resistance and the emf source are both variable. The graph of seven readings of the voltmeter and the ammeter (V and I, respectively) for different setting of resistance and the emf, taken at equal intervals of time Δt, are shown (on the right) by the dots connected by the curve EFGH. Consider the interval resistance of the battery to be negligible and the voltmeter and ammeter to be ideal devices. Take $R_0 \equiv V_0/I_0$. **[2017]**

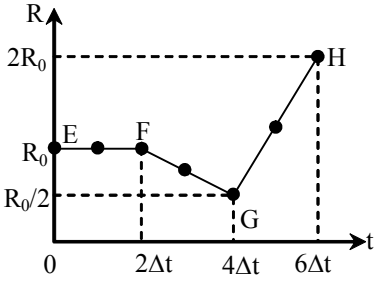


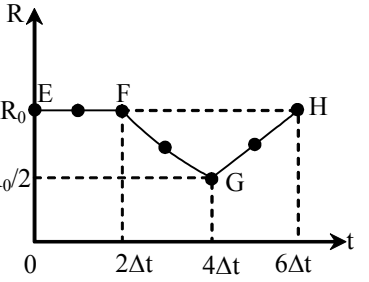


Then the plot of the resistance as a function of time corresponding to the curve EFGH is given by

(A)


(B)


(C)


(D)


Sol. **[D]**

From E $\rightarrow$ F

Slope is constant

V = IR

Thus R must be constant

$R_0 = V_0 / I_0$

From F $\rightarrow$ G

V = constant

I is increase thus R must be decrease.

at G, $R = \frac{V_0}{2I_0} = \frac{R_0}{2}$

From G $\rightarrow$ H

I = constant

V = increase, thus R must be increase

$R_H = \frac{2V_0}{2I_0} = R_0$

70. Stoke's law states that the viscous drag force F experienced by a sphere of radius a, moving with a speed V through a fluid with coefficient of viscosity η, is given by $F = 6\pi\eta a v$. If this fluid is flowing through a cylindrical pipe of radius r, length *l* and a pressure difference of P across its two ends, then the volume of water V which flows through the pipe in time t can be written as $\frac{V}{t} = k\left(\frac{P}{\ell}\right)^a \eta^b r^c$, where k is a dimensional constant.

Correct values of a, b and c are **[2017]**

(A) a = 1, b = –1, c = 4

(B) a = –1, b = 1, c = 4

(C) a = 2, b = –1, c = 3

(D) a = 1, b = –2, c = –4

Sol. **[A]**

$$\frac{V}{t} = k\left(\frac{P}{\ell}\right)^a . \eta^b . \gamma^c$$

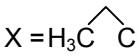
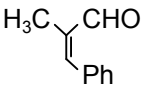
[r] = L; [l] = L
[P] = $ML^{-1}T^{-2}$; [η] = $ML^{-1}T^{-1}$
[V] = L^3
[t] = T
By calculation
a = 1; b = –1; c = 4

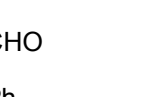
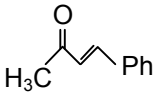
Section 7 - Part B- Chemistry

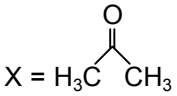
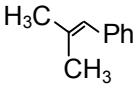
71. The reaction of an alkene X with bromine produce a compound Y, which has 22.22% C, 3.71% H and 74.07% Br. The ozonolysis of alkene X gives only one product. The alkene X is : **[2017]**
[Given : atomic mass of c = 12 ; H = 1; Br = 80]
(A) ethylene (B) 1-butene (C) 2-butene (D) 3-hexene

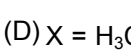
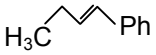
Sol. **[C]**
From the data given,
X will be CH_3–CH = CH–Ch_3 which on ozonolysis gives ethanal.

72. In the following reaction $H_3C - C \equiv C - H \xrightarrow[H_3O^+]{Hg^{2+}} X \xrightarrow[PhCHO]{dil.\ NaOH} Y$; X and Y, respectively, are **[2017]**

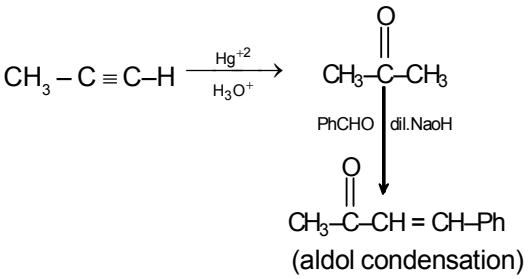
(A) X =  Y = 

(B) X =  Y = 

(C) X =  Y = 

(D) X =  Y = 

Sol. **[B]**



(aldol condensation)

73. $KMnO_4$ reacts with H_2O_2 in an acidic medium. The number of moles of oxygen produced per mole of $KMnO_4$ is **[2017]**
(A) 2.5 (B) 5 (C) 1.25 (D) 2

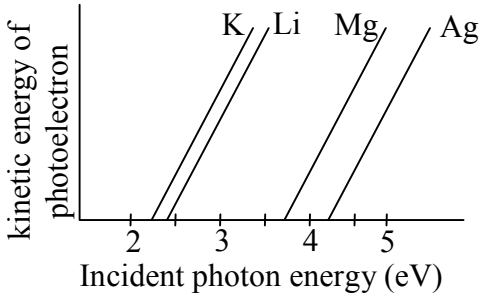
Sol. **[A]**

$(\text{no. of eq.})_{KMnO_4} = (\text{no. of eq.})_{H_2O_2}$

$\Rightarrow 1 \times 5 = x \times 2$

$\Rightarrow \boxed{x = 2.5}$

74. The photoelectric behaviour of K, Li, Mg and Ag metals is shown in the plot below. If light of wavelength 400 nm is incident on each of these metals, which of them will emit photoelectrons ? **[2017]**



(A) K (B) K and Li (C) K, Li and Mg (D) K, Li, Mg and Ag

Sol. **[B]**

If energy of incident light ≥ work function light

Then photoelectrons will be ejected.

75. A piece of metal weighing 100 g is heated to 80º C and dropped into 1 kg of cold water in an insulated container at 15ºC. If the final temperature of the water in the container is 15.69 ºC. If the final temperature of the water in the container is 15.69 ºC, the specific heat of the metal in J/g. ºC is **[2017]**

(A) 0.38 (B) 0.24 (C) 0.45 (D) 0.13

Sol. **[C]**

$Q = mS\Delta T$

Use the formula & get the answer

Section 8 Part-B Biology

76. The nucleus of a diploid organism contains 3 ng of DNA in G_1 phase. Which ONE of the following statements describes the state of the cell at the end of S phase ? **[2017]**

(A) The nucleus divides into two, and each nucleus contains 3 ng of DNA

(B) The nucleus does not divide, and it contains 3 ng of DNA

(C) The nucleus divides into two, and each nucleus contains 1.5 ng of DNA

(D)The nucleus does not divide and it contains 6 ng of DNA

Sol. **[D]**

In S-phase DNA replication occurs

77. Three cellular processes are listed below. Choose the Correct combination of processes that involve proton gradient across the membrane.

(i) Photosynthesis (ii) Aerobic respiration (iii) Anaerobic respiration **[2017]**

(A) ii and iii (B) i and ii (C) i, ii and iii (D) i and iii

Sol. **[B]**

In photosynthesis - photophosphorylation & in aerobic respiration - oxidative phosphorylation occurs that requires proton gradient.

78. The concentration of OH^- ions in a solution with the H^+ ions concentration of 1.3×10^{-4} M is **[2017]**

(A) 7.7×10^{-4} M (B) 1.3×10^{-4} M (C) 2.6×10^{-8} M (D) 7.7×10^{-11} M

Sol. **[D]**

$[H^+][OH^-] = 10^{-14}$

$1.3 \times 10^{-4} \times [OH] = 10^{-14}$

$$[OH^-] = \frac{1}{1.3} \times 10^{-10}$$
$$= 0.769 \times 10^{-10}$$
$$= 7.7 \times 10^{-11}$$

79. Given that tidal volume is 600 ml, inspiratory reserve volume is 2500 ml, and expiratory reserve volume is 800 ml, what is the value of vital capacity of lung ? **[2017]**

(A) 3900 ml (B) 3300 ml (C) 3100 ml (D) 1400 ml

Sol. **[A]**

VC = TV + IRV + ERV

Forcefully inspiration after forcefully expiration

80. Which of the following organisms produce sperm without involving meiosis ? **[2017]**

(A) Sand fly and fruit fly (B) House fly and grasshopper

(C) Honeybee and ant (D) Zebra fish and frog

Sol. **[C]**

Because they are haploid organism.