

# CAREER POINT

## MOCK TEST PAPER for IIT JEE (Advanced)

### PAPER-1 & 2

#### Physics, Chemistry & Mathematics

#### SOLUTION

#### PAPER -1

#### PHYSICS

Q.No.	1	2	3	4	5	6	7	8	9	10	
Ans.	B,C,D	B,C	A,C,D	A,D	A,B,D	B,D	A,B,C,D	A,B,C	A,C	A,D	
Numeric Response	Q.No.	1	2	3	4	5	6	7	8	9	10
	Ans.	4	3	3	5	5	4	5	8	9	1

#### CHEMISTRY

Q.No.	1	2	3	4	5	6	7	8	9	10	
Ans.	B	B,D	C,D	A,B,C,D	A,B,C,D	A,B,C,D	A,C,D	A,B	A,B,C	B,C	
Numeric Response	Q.No.	1	2	3	4	5	6	7	8	9	10
	Ans.	9	4	3	4	2	1	6	2	4	3

#### MATHEMATICS

Q.No.	1	2	3	4	5	6	7	8	9	10	
Ans.	A,B,D	B,C	A,C	A,C,D	B,D	A,B,C,D	B,C	A,B,C	C,D	B,C	
Numeric Response	Q.No.	1	2	3	4	5	6	7	8	9	10
	Ans.	2	1	4	5	3	2	1	4	3	4

#### PHYSICS

#### Section-I

1. [B,C,D]

at point 3

$$\frac{dv}{dr} = 0$$

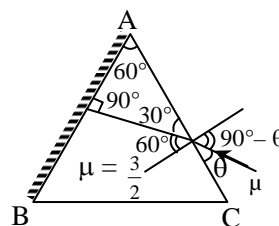
$$\therefore E = 0$$

Since near charge  $Q_2$ ,  $V$  is  $-ve$  while  $Q_1$  it is  $+ve$ , we can conclude  $Q_2$  and  $Q_1$  are negative and positive respectively.

To the right of  $Q_2$  the potential is  $+ve$  this implies that in the entire region to the right of  $Q_2$  the potential produced by  $Q_1$  is greater in absolute value than the potential produced by  $Q_2$

$$\therefore |Q_1| > |Q_2|$$

2. [B, C]



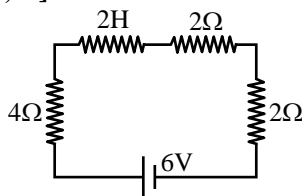
From snell's law

$$\mu \sin(90^\circ - \theta) = \frac{3}{2} \sin 60^\circ$$

$$\mu \cos \theta = \frac{3\sqrt{3}}{4}$$

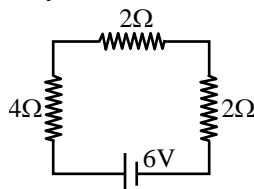
$$\theta = \cos^{-1} \left( \frac{3\sqrt{3}}{4\mu} \right)$$

3. [A, C, D]



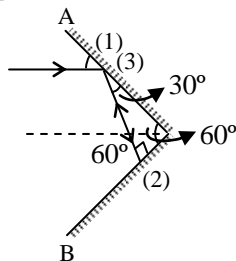
$$\tau = \frac{L}{R} = \frac{2}{8} = \frac{1}{4} \text{ sec}$$

in steady state



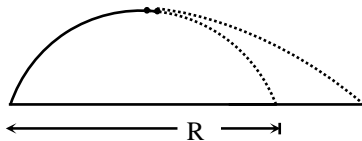
$$= \frac{6}{8} = \frac{3}{4} \text{ Amp} = 0.75 \text{ Amp.}$$

4. [A,D]



An mirror light fall normally. Thus light retrace and reflect at mirror A. Total no. of reflection is 3. Deviation of light is  $\pi$ .

5. [A,B,D]



By mass moment

$$m_1 \times x_1 = m_2 \times x_2$$

$$mR = 2m \times x_2$$

$$x_2 = R/2$$

Distance from launch point =  $3R/2$

$$R = \frac{u^2 \sin 2\theta}{g} = 108 \text{ m}$$

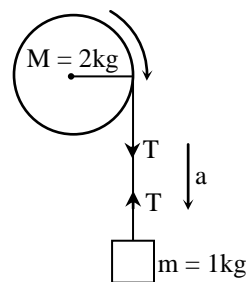
6. [B,D]

$$F_{\text{net}} = \frac{GMm}{4R^2} - \frac{GMm}{18R^2}$$

$$F_{\text{net}} = \frac{7}{36} \frac{GMm}{R^2}$$

7. [A,B,C,D]

By FBD of particle



$$mg - T = ma$$

$$10 - T = a$$

... (i)

By FBD of disc

$$TR = I\alpha = L \frac{a}{R} \Rightarrow T = \frac{MR^2}{2} \frac{a}{R^2}$$

$$T = Ma/2 = a$$

.... (ii)

By eq. (i) and (ii)

(A)  $a = 5 \text{ m/s}^2$  and  $T = 5\text{N}$  and  $\alpha = a/R = 5 \text{ rad/s}^2$

(B) For angular displacement of disc :

$$\theta = \omega t + 1/2 \alpha t^2$$

(C) Work done by torque

$$= \int \tau d\theta = \tau \int d\theta = 5 \times 40 = 200 \text{ J}$$

(D)  $\Delta \text{K.E.} = \Delta w = 200 \text{ J}$

$$K_2 - K_1 = 200 \text{ J}$$

8. [A,B,C]

(A) Heat required is -

$$Q = nC_p \Delta T = n(C_v + R)\Delta T$$

$$= \frac{1000}{28} [5 + 2] \times 120 = 30 \times 10^3 \text{ cal}$$

$$Q = 30 \text{ Kcal}$$

(B) The increase in the internal energy is

$$\Delta U = nC_v \Delta T = \frac{1000}{28} \times 5 \times 120$$

$$= 21 \text{ Kcal}$$

(C)  $Q = nC_v \Delta T + P\Delta V$

for constant volume  $\Delta V = 0$

$$Q = nC_v \Delta T = 21 \text{ Kcal}$$

(D) external work done is

$$W = Q - \Delta U = 8.6 \text{ KCal}$$

9. [A,C]

Maximum speed of any point on the string

$$= a\omega$$

$$= a(2\pi f)$$

$$\therefore \frac{v}{10} = \frac{10}{10} = 1 \quad (\text{Given : } v = 10 \text{ m/s})$$

$$\therefore 2\pi af = 1$$

$$\therefore f = \frac{1}{2\pi a}$$

$$a = 10^{-3} \text{ m}$$

(Given)

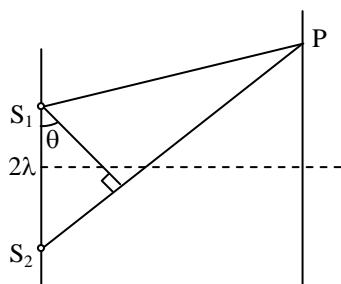
$$\therefore f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

Speed of wave,  $v = f\lambda$

$$\therefore (10 \text{ m/s}) = \left( \frac{10^3}{2\pi} \text{ s}^{-1} \right) \lambda$$

$$\therefore \lambda = 2\pi \times 10^{-2} \text{ m}$$

10. [A,D]



$$\delta = \frac{\pi}{2} - \frac{2\pi}{\lambda} (2\lambda \sin \theta) \Rightarrow \delta = \frac{\pi}{2} - 4\pi \sin \theta$$

For maxima,  $\delta = n\pi$

where  $n = 0, \pm 1, \pm 2 \dots$

$$\frac{\pi}{2} - 4\pi \sin \theta = n\pi$$

$$\sin \theta = \frac{\frac{1}{2} - n}{4}$$

$$n = 0, \quad \sin \theta = \frac{1}{8}$$

$$n = \pm 1, \quad \sin \theta = -\frac{1}{8}, +\frac{3}{8}$$

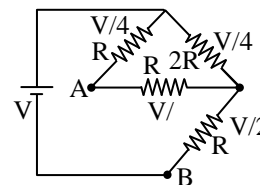
$$n = \pm 2, \quad \sin \theta = -\frac{3}{8}, \frac{5}{8}$$

$$n = \pm 3, \quad \sin \theta = -\frac{5}{8}, \frac{7}{8}$$

$$n = 4, \quad \sin \theta = -\frac{7}{8}$$

### Section-III

1. [4]



$$V_A - V_B = \frac{V}{4} + \frac{V}{2} \Rightarrow \frac{3V}{4}$$

$$\therefore U_{\text{stored}} = \frac{1}{2} \times 10 \times 10^{-3} \times \left( \frac{3V}{4} \right)^2$$

$$\Rightarrow \frac{4.5}{10} = \frac{5 \times 10^{-3} \times 9 \times V^2}{16}$$

$$V^2 = \frac{16}{10 \times 10^{-3}} \Rightarrow 1600$$

$$V = 40 \text{ volt}$$

2. [3] Range will become twice of velocity of efflux becomes twice. Now as,

$$v = \sqrt{2gh}$$

Therefore,  $h$  should become 4 times or 40 m

Thus, an extra pressure equivalent to 30 m of water should be applied.

$$1 \text{ atm} = 0.76 \times 13.6 \text{ m of water}$$

$$= 10.336 \text{ m of water}$$

$$30 \text{ m of water} \approx 3.0 \text{ atm}$$

3. [3]  $R = \frac{\ell}{KA}$

When heat is transferred from first vessel to second, temperature of first vessel decreases while that of second vessel increases. Due to both these reasons, difference between temperature of vessels decreases.

Let at an instant  $t$ , the temperature difference between two vessels be  $\theta$ .

$$H = \frac{\theta}{R} = \frac{KA\theta}{\ell}$$

$$dQ = Hdt = \frac{KA\theta}{\ell} dt \quad \dots(i)$$

Since gases are contained in two vessels, therefore, processes on gases in two vessels are isochoric.

Hence, decrease in temperature of gas in first vessel,

$$\Delta\theta_1 = \frac{dQ}{nC_v} = \frac{dQ}{2 \times \frac{5R}{2}} = \frac{dQ}{5R}$$

Increase in temperature of gas in second vessel is

$$\Delta\theta_2 = \frac{dQ}{4 \times \frac{3R}{2}} = \frac{dQ}{6R}$$

∴ Decrease in temperature difference

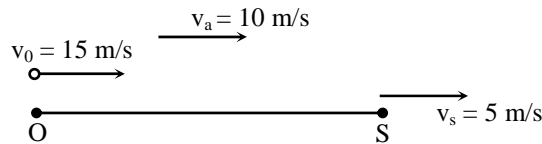
$$(-d\theta) = \Delta\theta_1 + \Delta\theta_2$$

$$-d\theta = \frac{dQ}{R} \times \frac{11}{30}$$

$$\text{or } - \int_{50}^{25} \frac{d\theta}{\theta} = \frac{KA \times 11}{30 \ell R} \int_0^t dt$$

$$t = 3 \text{ seconds.}$$

4. [5]



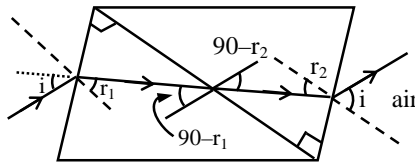
$$\therefore v_{\text{observer, air}} = 5 \text{ m/s}$$

$$v_{\text{source, air}} = -5 \text{ m/s}$$

$$\Rightarrow v = \frac{330+5}{330-5} \times 325 = 335 \text{ Hz}$$

$$\therefore v - 330 = 5 \text{ Hz}$$

5. [5]



$$\sqrt{3} \sin i = \sqrt{3} \sin r_1 \quad \dots(1)$$

$$\sqrt{3} \sin (90 - r_1) = \sqrt{2} \sin (90 - r_2)$$

$$\sqrt{3} \cos r_1 = \sqrt{2} \cos r_2 \quad \dots(2)$$

$$\sin i = \sqrt{2} \sin r_2 \quad \dots(3)$$

$$i = r_1$$

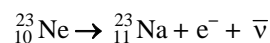
$$3 \cos^2 r_1 + \sin^2 i = 2$$

$$2 \cos^2 r_1 + \cos^2 r_1 + \sin^2 r_1 = 2$$

$$\cos^2 r_1 = \frac{1}{2}$$

$$r_1 = 45^\circ \therefore i = 45^\circ$$

6. [4]



$$Q = [m({}^{23}\text{Ne}) - m({}^{23}\text{Na})] \times 931.5 \text{ MeV}$$

$$A = 4.375 \text{ MeV} = 4.4 \text{ MeV}$$

$$Q \approx 4 \text{ MeV}$$

$$Q = KE_y + KE_e + E_{\bar{\nu}}$$

$$KE_y \text{ is very very small}$$

$$A \approx KE_e + E_{\bar{\nu}}$$

when  $KE_e$  is maximum  $E_{\bar{\nu}}$  is negligible

$$KE_e \approx Q = 4 \text{ MeV}$$

7. [5] The energy of the electron in the  $n^{\text{th}}$  state of  $\text{He}^+$  ion of atomic number  $Z$  is given by

$$E_n = - (13.6) \text{ eV} \frac{Z^2}{n^2} \text{ for } \text{H}^+ \text{ ion } Z = 2.$$

Therefore

$$E_n = - \frac{(13.6\text{eV}) \times (2)^2}{n^2} = - \frac{54.4}{n^2} \text{ eV}$$

The energies  $E_1$  and  $E_2$  of the two emitted photons in eV are

$$E_1 = \frac{12431}{1085} \text{ eV} = 11.4 \text{ eV}$$

$$\text{and } E_2 = \frac{12431}{304} \text{ eV} = 40.9 \text{ eV}$$

$$\text{Thus total energy } E = E_1 + E_2 = 11.4 + 40.9 = 52.3 \text{ eV}$$

Let  $n$  be the principle quantum number of excited state.

Now we have for the transition from  $n = n$  to  $n = 1$

$$E = - (54.4) \text{ eV} \left( \frac{1}{1^2} - \frac{1}{n^2} \right)$$

But  $E = 52.3 \text{ eV}$ . Therefore

$$52.3 \text{ eV} = 54.4 \text{ eV} \times \left( \frac{1}{1^2} - \frac{1}{n^2} \right)$$

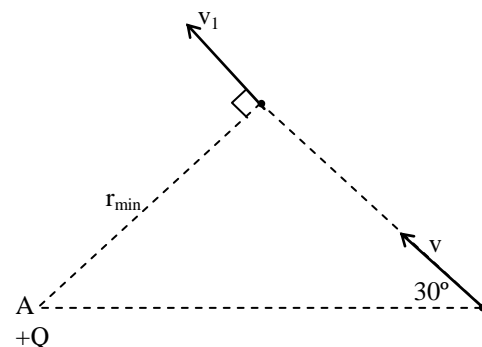
$$\text{or } 1 - \frac{1}{n^2} = \frac{52.3}{54.4} = 0.96$$

Which gives  $n^2 = 25$  or  $n = 5$

The energy of the incident electron = 100 eV (given). The energy supplied to  $\text{He}^+$  ion = 52.3 eV.

Therefore, the energy of the electrons left after the collision =  $100 - 52.3 = 47.7 \text{ eV}$ .

8. [8]



# CHEMISTRY

## Section – I

Angular momentum about A is conserved as

$$\tau_{\text{external}} = 0$$

$$mv \sin 30^\circ \times R = v_1 \times m \times r_{\text{min}}$$

$$\frac{Rv}{2} = \frac{1}{\sqrt{3}} \times r_{\text{min}}$$

$$r_{\text{min}} = \frac{\sqrt{3}R}{2} = \frac{1.713 \times 10}{2} = 8.665 = 8 \text{ cm}$$

9. [9] At y according to Kirchoff's junction law

$$\frac{y-x}{2} + \frac{y-x-100}{2} + \frac{y-50}{2} + \frac{y}{2} + \frac{y-50}{2} = 0$$

$$5y - 2x = 200 \quad \dots(1)$$

similarly at x

$$i = \frac{50-x}{2} + \frac{y-x}{2} \quad \dots(2)$$

at x + 100

$$i = \frac{x+100-50}{2} + \frac{x+100-y}{2} \quad \dots(3)$$

$$\text{we get } y - 2x = 50 \quad \dots(4)$$

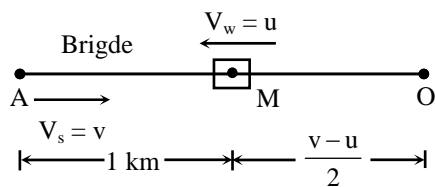
from (1) and (4)

$$y = 37.5 \text{ V}$$

So current through R is 18.75 A.

10. [1] Let  $V_w = u$  &  $U_{sw} = v$

Time taken by swimmer to go from M to O and O to B = time taken by float to reach B from M.



$$= \frac{1}{2} + \frac{1 + \frac{v-u}{2}}{v+u} = \frac{1}{u}$$

$$\Rightarrow \frac{1}{2} + \frac{2+v-u}{2(v+u)} = \frac{1}{u}$$

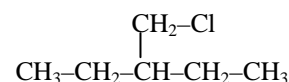
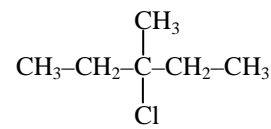
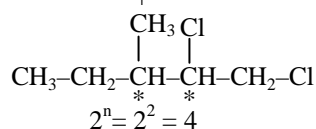
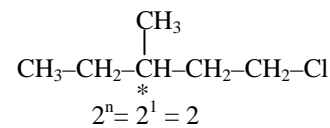
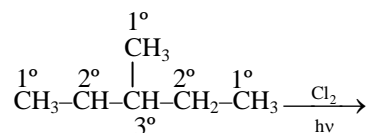
$$\Rightarrow \frac{(v+u+2+v-u)}{2(v+u)} = \frac{1}{u}$$

$$\Rightarrow (2v+2)u = 2(v+u)$$

$$\Rightarrow 2vu + 2u = 2v + 2u$$

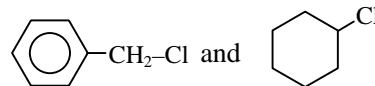
$$u = 1 \text{ km/hr}$$

2. [B,D]



Total (8) Product

3. [C,D] Because



have any chiral -C so that they give same product by  $\text{SN}^1$ ,  $\text{SN}^2$

4. [A,B,C,D]

All contains symmetry, with chiral centre

5. [A,B,C,D]

Theory

6. [A,B,C,D]

After dilution  $[\text{H}^+] = 10^{-2} \Rightarrow \text{pH} = 2$

Let V litre solution of  $\text{pH} = 2$  is added in original solution so that pH remains fixed.

$$\therefore [\text{H}^+] = \frac{10^{-2}x + V \times 10^{-2}}{10 + V} = 10^{-2}$$

This result is independent of volume taken.

7. [A,C,D]

$$x_A P_A^\circ + x_B P_B^\circ = 700 \quad \dots (i)$$

$$x_A P_A^\circ + x_B P_B^\circ = 0.30 P_A^\circ + 0.70 P_B^\circ = 600 \quad \dots (ii)$$

if moles of A & B initially are x & y then

$$x = 0.75 \times \frac{2}{3} (x+y) + 0.30 \times \frac{1}{3} (x+y)$$

$$\& \quad x_A = \frac{x}{x+y} \quad \text{or} \quad x_B = \frac{y}{x+y}$$

solving gives

$$x_A = 0.6, \quad x_B = 0.4, \quad P_A^0 = \frac{2500}{3} \text{ torr}$$

$$\& \quad P_B^0 = 500 \text{ torr}$$

8.[A,B]  $k_1 = k_2$

$$\Rightarrow \frac{2}{3} \text{ rd A has reacted for } [A] = [C] = [D]$$

$$\therefore k_1 + k_2 = \frac{1}{t} \ln \frac{[A]_0}{\frac{1}{3}[A]_0}$$

$$\Rightarrow t = \frac{1}{k_1 + k_2} \ln 3 = \frac{1}{2k_1} \ln 3 = \frac{1}{2k_2} \ln 3$$

9.[A,B,C]



In each mole of  $MCl_x$  there are  $x$  moles of  $Cl^-$

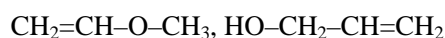
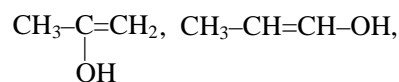
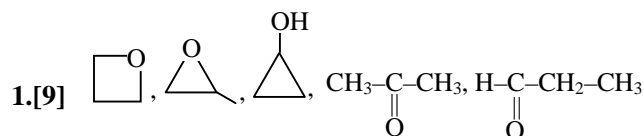
$$\Rightarrow [Cl^-] = x \times 0.01$$

conc. of  $[M^{x+}] = 0.01$

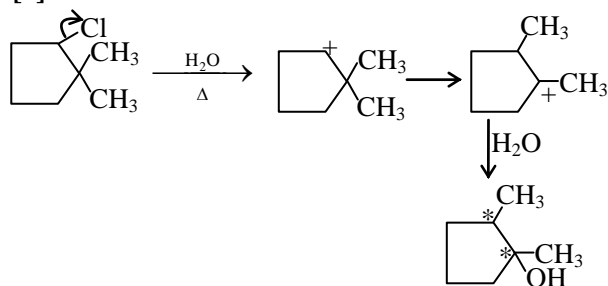
10.[B,C]

the pressure of  $NH_3$  will decrease due to addition of  $CO_2$  (backward, shifting Le-chatelies's principle. The pressure of  $CO_2$  will be more than 0.1 atm.

### Section - III



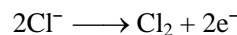
2.[4]



$$S.I. = 2^2 = 4$$

4.[4] (I, II, V, VI are correct)

5.[2]



Moles of  $NaClO$  required

$$= \frac{10^6 \times 1 \times 7.45/100}{74.5} = 10^3 \text{ mol}$$

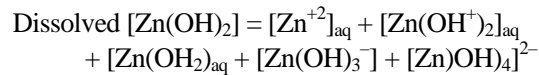
moles of  $Cl_2$  required =  $10^3$

eq. of  $Cl_2$  required =  $2 \times 10^3$

$$2 \times 10^3 \times 96500 = 9.65 \times t$$

$$2 \times 10^7 = t$$

6.[1]



Now,  $[Zn(OH)_2]_{aq} = 10^{-6} \text{ M}$  in saturated solution.

$$\text{so, } [Zn(OH)^+] = \frac{10^{-6} \times 10^{-7}}{[OH^-]} = \frac{10^{-13}}{[OH^-]}$$

$$\text{Similarly, } [Zn^{+2}] = \frac{10^{-17}}{[OH^-]^2},$$

$$[Zn(OH)_3] = 10^{-3} [OH^-],$$

$$[Zn(OH)_4]^{2-} = K_5 [Zn(OH)_3^-]$$

$$[OH^-] = (10^{-2} \text{ M}^{-1}) [OH^-]^2$$

$$\text{Dissolved } Zn(OH)_2 = \frac{10^{-17}}{[OH^-]^2} + \frac{10^{-13}}{[OH^-]}$$

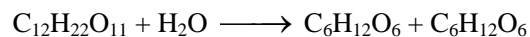
$$+ 10^{-6} + 10^{-3} \times [OH^-] + 10^{-2} [OH^-]^2$$

$$= \frac{10^{-17}}{10^{-16}} + \frac{10^{-13}}{10^{-8}} + 10^{-6} + 10^{-3} \times 10^{-8} + 10^{-18}$$

$$= 10^{-1} + 10^{-5} + 10^{-6} + 10^{-11} = 10^{-1}$$

$$= -\log Zn(OH)_2 (aq) = 1$$

7.[6]



mol	0.0125	0	0
	0.0125 - x	x	x

$$\Delta T_b = m_1 K_b + m_2 K_b + m_3 K_b$$

$$m_1 + m_2 + m_3 = \frac{0.104}{0.52} = 0.2$$

$$\frac{0.125 - x + x + x}{100} \times 100 = 60$$

$$x = 0.0075$$

$$\text{mol \%} = \frac{0.0075}{0.0125} \times 100 = 60$$

$$\frac{1}{10} \text{ th of mol\%} = \frac{60}{10} = 6$$

8.[2]

$$2.303 \log K = -\frac{E_a}{RT} + 2.303 \log A$$

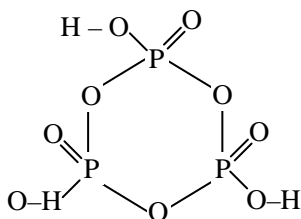
$$\text{Thus, } -\frac{E_a}{2.303R} = \tan \theta = -\frac{1}{2.303}$$

$$\therefore E_a = R = 2 \text{ Cal.}$$

$$9.[4] \text{ \% of carbon} = \frac{144}{144 + m + 35.5(10 - m)} \times 100 = 40$$

On solving  $m = 4$

10.[3]



## MATHEMATICS

### Section - I

1.[A, B, D]

$$f(x) = \begin{cases} \frac{1}{x+1}, & 0 \leq x < 1 \\ \frac{x}{2}, & 1 \leq x < 2 \\ \frac{x}{x-1}, & 2 \leq x < \frac{5}{2} \end{cases}$$

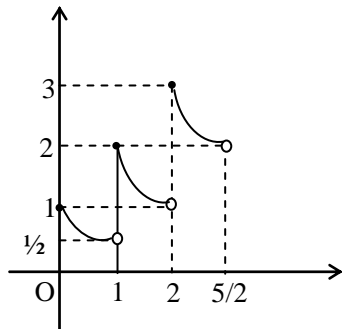
Clearly  $f(x)$  is discontinuous and bijective function

$$\lim_{x \rightarrow 1^-} f(x) = \frac{1}{2}$$

$$\lim_{x \rightarrow 1^+} f(x) = 2$$

$$\min(\lim_{x \rightarrow 1^-} f(x), \lim_{x \rightarrow 1^+} f(x)) = \frac{1}{2} \neq f(1)$$

$$\text{Max}(1, 2) = 2 = f(1)$$



2.[B,C] From hyperbola & conjugate hyperbola

$$\frac{1}{e^2} + \frac{1}{(f(e))^2} = 1$$

$$\Rightarrow f(e) = \frac{e}{\sqrt{e^2 - 1}}$$

$$f(f(f(e))) = \frac{e}{\sqrt{e^2 - 1}}$$

$$\underbrace{fff \dots f(e)}_{n \text{ times}} = \begin{cases} e, & n \text{ is even} \\ \frac{e}{\sqrt{e^2 - 1}}, & n \text{ is odd} \end{cases}$$

$n$  is even

$$\int_1^3 \underbrace{fff \dots f(e)}_{n \text{ times}} de = \int_1^3 e de = \left. \frac{e^2}{2} \right|_1^3 = 4$$

$n$  is odd

$$\int_1^3 \underbrace{fff \dots f(e)}_{n \text{ times}} de = \int_1^3 \frac{e}{\sqrt{e^2 - 1}} de = \left. \sqrt{e^2 - 1} \right|_1^3 = \sqrt{8} - 0 = 2\sqrt{2}$$

3.[A,C]

Equation of tangent at  $P(4 \cos \theta, 2 \sin \theta)$  is

$$x \cos \theta + 2y \sin \theta = 4$$

It pass through  $(4, 2)$

$$\therefore \sin \theta + \cos \theta = 1$$

$$\theta = 0, \frac{\pi}{2}$$

4.[A,C,D]

$$\sin \left( x \frac{dy}{dx} - y \right) = \frac{dy}{dx}$$

$$x \frac{dy}{dx} - y = \sin^{-1} \frac{dy}{dx} \quad \dots(i)$$

again diff.

$$\frac{dy}{dx} + \frac{x d^2y}{dx^2} - \frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(\frac{dy}{dx}\right)^2}} \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} \left( x - \frac{1}{\sqrt{1 - \left(\frac{dy}{dx}\right)^2}} \right) = 0$$

$$\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{dy}{dx} = c$$

from equation (i)

$$cx - y = \sin^{-1} c$$

$$c = 0, \quad y = 0$$

$$\text{or } x = \frac{1}{\sqrt{1 - \left(\frac{dy}{dx}\right)^2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = 1 - \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2 - 1}}{x}$$

from equation (i)

$$\frac{x\sqrt{x^2 - 1}}{x} - y = \sin^{-1} \frac{\sqrt{x^2 - 1}}{x}$$

$$y = \sqrt{x^2 - 1} - \sin^{-1} \frac{\sqrt{x^2 - 1}}{x}$$

**5.[B,D]**  $f(x) = x^2 + ax^2 + bx^3 = (a+1)x^2 + bx^3$

$$a = \int_{-1}^1 t f(t) dt = \int_{-1}^1 (a+1)t^3 + bt^4 dt$$

$$= 2b \int_0^1 t^4 dt$$

$$a = \frac{2b}{5} \quad \dots(1)$$

$$b = \int_{-1}^1 f(t) dt = \int_{-1}^1 (a+1)t^2 + bt^3 dt$$

$$= \frac{(a+1)t^3}{3} \Big|_{-1}^1$$

$$b = (a+1) \frac{2}{3} \quad \dots(2)$$

$$a = \frac{4}{11} \quad b = \frac{10}{11}$$

$$f(x) = \frac{15x^2}{11} + \frac{10x^3}{11}$$

$$f(1) = \frac{15}{11} + \frac{10}{11} = \frac{25}{11}$$

$$f(-1) = \frac{15}{11} - \frac{10}{11} = \frac{5}{11}$$

$$f(1) + f(-1) = \frac{30}{11}$$

$$f(1) - f(-1) = \frac{20}{11}$$

**6.[A,B,C,D]**

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

angle between  $\vec{b}$  and  $\vec{c}$

= angle between  $\vec{a}$  and  $\vec{c}$

$$\Rightarrow \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = \cos \alpha$$

$$(\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1)$$

$$\text{also } \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$\text{so } \vec{c} = \ell \vec{a} + m \vec{b} + n(\vec{a} \times \vec{b})$$

$$\therefore \vec{a} \cdot \vec{c} = \ell = \cos \alpha, \vec{b} \cdot \vec{c} = \cos \alpha = m$$

$$\text{Now } |\vec{c}| = 1 \Rightarrow \vec{c} \cdot \vec{c} = 1 = 2\ell^2 + n^2 + |\vec{a} \times \vec{b}|^2$$

$$\Rightarrow m = \ell = \cos \alpha$$

$$\Rightarrow 1 = 2\ell^2 + n^2$$

$$\Rightarrow n^2 = 1 - 2\ell^2 = 1 - 2\cos^2 \alpha = -\cos 2\alpha$$

$$\Rightarrow \ell^2 = m^2 = \frac{1-n^2}{2} = \frac{1+\cos 2\alpha}{2}$$

Hence all are correct.

**7.[B, C]**  $x^2 + 400 = y^2 \quad \dots(1)$

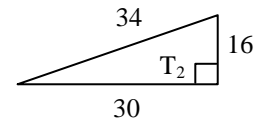
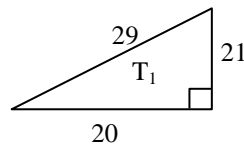
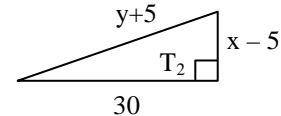
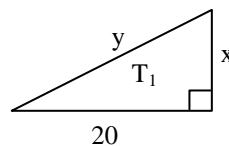
$$\& 900 + (x-5)^2 = (y+5)^2$$

$$\Rightarrow 900 + x^2 - 10x = y^2 + 10y$$

$$\Rightarrow 900 + x^2 - y^2 = 10(x+y)$$

$$\Rightarrow 500 = 10(x+y)$$

$$\Rightarrow x+y = 50 \quad \dots(2)$$



Form (1) & (2)

$$y - x = 8$$

From (2) & (3)  $\dots(3)$

So,  $y = 29$  &  $x = 21$

$$\therefore \Delta_1 = 210 \text{ \& } \Delta_2 = 240$$

$$\therefore \frac{\Delta_2}{\Delta_1} = \frac{8}{7} \Rightarrow 8\Delta_1 = 7\Delta_2$$

$$\text{Also, } P_1 = \frac{\Delta_1}{S} = \frac{210}{35} = 6$$

$$\& P_2 = \frac{\Delta_2}{S} = \frac{240}{40} = 6$$

**8.[A,B,C]**

Clearly O is the mid point of SS' and HH'

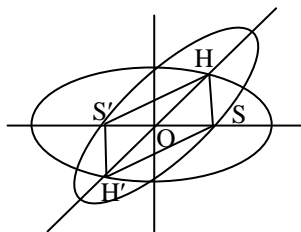
$\Rightarrow$  diagonals of quadrilateral HSH'S' bisect each other so, it is a parallelogram.

$$\text{Let } H'O'H = 2r \Rightarrow OH = r = ae_2$$



H lies on  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (suppose)

$$\therefore \frac{r^2 \cos^2 \theta}{a^2} + \frac{r^2 \sin^2 \theta}{b^2} = 1$$



$$e_2^2 \cos^2 \theta + \frac{e_2^2 \sin^2 \theta}{1 - e_1^2} = 1 \quad (\because b^2 = a^2(1 - e_1^2))$$

$$e_2^2 \cos^2 \theta - \frac{e_2^2 \cos^2 \theta}{1 - e_1^2} = 1 - \frac{e_2^2}{1 - e_1^2}$$

$$\Rightarrow \cos^2 \theta = \frac{1}{e_1^2} + \frac{1}{e_2^2} - \frac{1}{e_1^2 e_2^2}$$

if  $\theta = 90^\circ$

$$\frac{e_1^2 + e_2^2}{e_1^2 e_2^2} = \frac{1}{e_1^2 e_2^2} = e_1^2 + e_2^2 = 1$$

9.[C,D]  $\lim_{n \rightarrow \infty} \tan(1/n) \ln(1/n)$

$$= \lim_{n \rightarrow \infty} \frac{\tan(1/n)}{(1/n)} \cdot \frac{\ln(1/n)}{n}$$

$$= - \lim_{n \rightarrow \infty} \frac{\tan(1/n)}{(1/n)} \cdot \frac{\ln(n)}{n}$$

$$= -1 \cdot \lim_{n \rightarrow \infty} \frac{1/n}{1} = 0$$

then,  $f(x) = e^0 = 1$

$$\therefore \int \frac{f(x)}{\sqrt[3]{(\sin^{11} x \cos x)}} dx = \int \frac{1}{\sin^{11/3} x \cos^{1/3} x} dx$$

$$= \int \sin^{-11/3} x \cdot \cos^{-1/3} x dx$$

=

$$\int (\tan x)^{-11/3} \cos^{-4} x dx = \int (\tan x)^{-11/3} \cdot \cos^{-4} x dx$$

$$= \int (\tan x)^{-11/3} \cdot (1 + \tan^2 x) \cdot \sec^2 x dx$$

$$= \frac{(\tan x)^{-11/3+1}}{\left(\frac{-11}{3}+1\right)} + \frac{(\tan x)^{-2/3}}{(-2/3)} + c$$

$$= -\frac{3}{8} (\tan x)^{-8/3} - \frac{3}{2} (\tan x)^{-2/3} + c$$

$$\therefore g(x) = -\frac{3}{8} (\tan x)^{-8/3} - \frac{3}{2} (\tan x)^{-2/3}$$

$$\therefore g(\pi/4) = -\frac{3}{8} - \frac{3}{2} = -\frac{15}{8}$$

and  $g(x)$  is non differentiable at  $\tan x = 0$

or  $x = n\pi, n \in \mathbb{I}$

10.[B,C] Given  $f_6(x) = \frac{x-1}{x}$  ... (1)

$$f_6(f_m(x)) = f_4(x) = \frac{1}{1-x} \quad (\text{given})$$

$$\therefore f_6(f_m(x)) = \frac{f_m(x)-1}{f_m(x)} = \frac{1}{1-x}$$

(using given relation  $f_6(x) = \frac{x-1}{x}$ )

put  $f_m(x) = k$

$$\frac{k-1}{k} = \frac{1}{1-x}$$

$$k - kx - 1 + x = k \Rightarrow k = \frac{x-1}{x}$$

$$\Rightarrow f_m(x) = \frac{x-1}{x} = f_6(x) \Rightarrow m = 6$$

again  $f_n(f_4(x)) = f_3(x) = \frac{1}{x}$

$$f_n\left(\frac{1}{1-x}\right) = \frac{1}{x}; \text{ let } \frac{1}{1-x} = t$$

$$\Rightarrow t - tx = 1 \Rightarrow x = \frac{t-1}{t}$$

$$\therefore f_n(t) = \frac{t}{t-1} \Rightarrow f_n(x) = \frac{x}{x-1} = f_5(x)$$

hence  $n = 5$

### Section - III

1.[2] Volume of the parallelepiped formed by

$\vec{a}', \vec{b}', \vec{c}'$  is 4

$\therefore$  Volume of the parallelepiped formed by  $\vec{a},$

$\vec{b}, \vec{c}$  is  $\frac{1}{4}$

$$\vec{b} \times \vec{c} = \frac{(\vec{c}' \times \vec{a}') \times \vec{c}}{4} = \frac{1}{4} \vec{a}'$$

$$\therefore |\vec{b} \times \vec{c}| = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$$\therefore \text{length of altitude} = \frac{1}{4} \times 2\sqrt{2} = \frac{1}{\sqrt{2}}$$

$$2.[1] \quad AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

If  $AB = BA$ , then  $a + 2c = a + 3b$   
 $\Rightarrow 2c = 3b \Rightarrow b \neq 0$   
 $b + 2d = 2a + 4b \Rightarrow 2d - 2a = 3b$

$$\frac{d-a}{3b-c} = \frac{\frac{3}{2}b}{3b-\frac{3}{2}b} = 1$$

3.[4] 
$$\frac{E(1) + E(2) + E(3) + \dots + E(100)}{100}$$
  

$$= \frac{E(010203 \dots 99.100)}{100}$$
  

$$= \frac{20(2+4+6+8)}{100} = \frac{400}{100} = 4$$

4.[5] Highest power of  $x$  in numerator & denominator is 2010

$$\therefore \lim_{x \rightarrow \infty} \frac{x^{2010} \sum_{r=1}^{10} \left(1 + \frac{r}{x}\right)^{2010}}{x^{1006} \left(1 + \frac{1}{x^{1006}}\right) x^{1004} \left(2 + \frac{1}{x^{1004}}\right)}$$

$$= \frac{\sum_{r=1}^{10} (1)}{(1+0)(2+0)} = \frac{10}{2} = 5$$

5.[3] 
$$\lim_{x \rightarrow 0} \frac{\sin x^4 - x^4 \cos x^4 + x^{20}}{x^4 (e^{2x^4} - 1 - 2x^4)}$$
  

$$= \lim_{t \rightarrow 0} \frac{\sin t - t \cos t + t^5}{t(e^{2t} - 1 - 2t)}$$
  

$$= \lim_{t \rightarrow 0} \frac{t - \frac{t^3}{3!} + \frac{t^5}{5!} \dots - t \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} \dots\right) + t^5}{t \left(1 + 2t + \frac{4t^2}{2!} + \frac{8t^3}{3!} + \frac{16t^4}{4!} + \dots - 1 - 2t\right)}$$
  

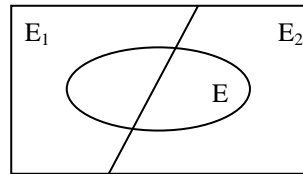
$$= \lim_{t \rightarrow 0} \frac{-\frac{t^3}{6} + \frac{t^3}{2} + \frac{t^5}{5!} - \frac{t^5}{4!} + \dots + t^5}{2t^3 + \frac{8t^4}{3!} + \dots}$$
  

$$= \frac{-\frac{1}{6} + \frac{1}{2}}{2} = \frac{-1+3}{12} = \frac{1}{6}$$

6.[2]

UrnA  $\begin{cases} 9R \\ 11W \end{cases}$  ; UrnB  $\begin{cases} 12R \\ 3W \end{cases}$

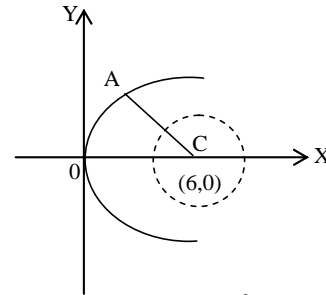
$E$  : event of drawing a red ball;  
 $E_1 = 1$  or 2 on die  
 $E_2 = 3, 4, 5, 6$  on die  
 $E = (E \cap E_1) + (E \cap E_2)$   
 $P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$   
 Using the law of total probabilities,



$$P(\text{red ball}) = \frac{2}{6} \cdot \frac{9}{20} + \frac{4}{6} \cdot \frac{12}{15} = \frac{41}{60}$$

7.[1]  $f(1) = -6$   
 For maximum at  $x = 1$   
 $\lim_{x \rightarrow 1^-} f(x) = \tan^{-1} \alpha - 5 < -6$   
 $\Leftrightarrow \tan^{-1} \alpha < -1 \Leftrightarrow \alpha < -\tan 1$

8.[4]



Any normal of parabola  $y^2 = 4x$  is  
 $y = -tx + 2t + t^3$   
 if it pass through  $(6, 0)$ , then  
 $-6t + 2t + t^3 = 0$   
 $\Rightarrow t = 0$ ;  $t^2 = 4$   
 $\therefore A(4, 4)$   
 $\therefore$  for no common tangent  $AC > r$   
 $\Rightarrow \sqrt{20} > r \Rightarrow r < \sqrt{20}$

9.[3] Let  $y = \frac{e^x - e^{-x}}{2} \Rightarrow e^{2x} - 1 = 2ye^x$   
 Therefore,  $t^2 - 2yt - 1 = 0$ ,  $t = e^x$   
 $\Rightarrow t = \frac{2y \pm \sqrt{4y^2 + 4}}{2} = y \pm \sqrt{y^2 + 1}$   
 $\Rightarrow x = \log \left( y + \sqrt{y^2 + 1} \right)$  (Since  $e^x > 0$ )  
 $\therefore f^{-1}(x) = g(x) = \log \left( x + \sqrt{x^2 + 1} \right)$   
 $g \left( \frac{e^{1002} - 1}{2e^{501}} \right) = \log \left( \frac{e^{1002} - 1}{2e^{501}} + \frac{e^{1002} + 1}{2e^{501}} \right)$   
 $= \log e^{501} = 501$

10.[4] Equation of plane  $\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$   
 $\Rightarrow 11x - y - 3z = 28$   
 $m = -1, n = -3$   
 $|m + n| = 4$

**PAPER -2**

**PHYSICS**

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	A	D	B	D	D	D	A	D	C	D
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	D	D	A	C	A	B	C	C

**CHEMISTRY**

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	D	D	C	C	B	D	C	B	C	C
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	A	C	C	B	A	C	B	D

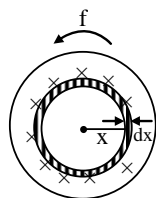
**MATHEMATICS**

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	D	A	A	A	A	D	B	B
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	C	D	B	A	A	C	C	A	D	B

**PHYSICS**

**Section-I**

1. [A]



Charge of element at ring  $dq = \sigma (2\pi x dx)$

$$dB = \frac{\mu_0 dI}{2x} \Rightarrow dB = \frac{\mu_0}{2x} \frac{dq \omega}{2\pi}$$

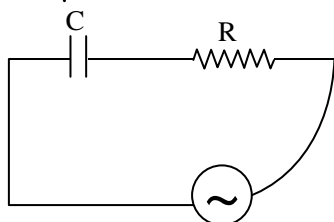
$$B = \int dB = \frac{\mu_0 \omega}{4\pi} \int_0^R \frac{\sigma 2\pi x dx}{x}$$

$$B = \frac{\mu_0 \omega \sigma R}{2} = \frac{\mu_0 2\pi f q R}{2\pi R^2} = \frac{\mu_0 q f}{R}$$

2. [D]

$$V_L = V_C = V_R,$$

$$V_{\text{source}} = \sqrt{10^2 + (10 - 10)^2} = 10 \text{ V}$$



$$\therefore ix_L = ix_C = iR$$

$$\therefore x_L = x_C = R$$

$$\therefore V_c = ix_C$$

$$= \frac{V_{\text{source}}}{\sqrt{R^2 + x_C^2}} x_C$$

$$= \frac{V_{\text{source}} R}{\sqrt{R^2 + R^2}} = \frac{10V}{\sqrt{2}}$$

3. [B] energy = power  $\times$  time =  $10^6 \times 24 \times 60 \times 60$

$$= 24 \times 36 \times 10^8 \text{ Joule}$$

$$\text{No. of uranium atoms used} = \frac{24 \times 36 \times 10^8}{32 \times 10^{-12}}$$

$$\therefore \text{required mass} = \frac{235}{6 \times 10^{23}} \times 27 \times 10^{20}$$

$$\approx 1 \text{ gm}$$

4. [D]  $mv = (m + M)V_x \Rightarrow V_x = \frac{mv}{(m + M)}$

From energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}(m + M)V_x^2 + \frac{1}{2}mV_y^2 + mgh_1$$

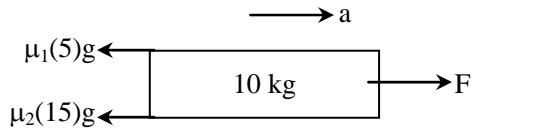
$$V_y^2 = v^2 - \frac{mv^2}{m + M} - 2gh_1$$

If  $h_2$  is the height from break off point then

$$h_2 = \frac{V_y^2}{2g}$$

$$\& h = h_1 + h_2 = \frac{Mv^2}{2g(m+M)}$$

5. [D]  $(F + 80N) - \mu_2(5 \text{ kg} + 10 \text{ kg})g = (15 \text{ kg})a$   
 $\frac{F + 80 - 60}{15} = a \quad \dots(1)$

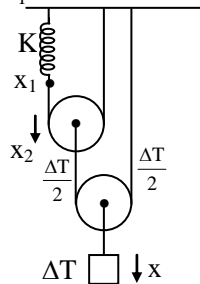


$F - \mu_2(15) \times 10 - \mu_1(5 \times 10) = 10 \times a$   
 $\frac{F - 90}{10} = a \quad \dots(2)$

So, we can write  
 $\frac{F + 20}{15} = \frac{F - 90}{10}$   
 $F = 310 \text{ N}$

6. [D]  $R = u_{xt} + \frac{1}{2} a_x t^2$   
 $= u \cos \theta \times \frac{2u \sin \theta}{g} + \frac{1}{2} \left( \frac{g}{4} \right) \left( \frac{2u \sin \theta}{g} \right)^2$   
 $= \frac{2u^2 \sin \theta \cos \theta}{g} + \frac{(u \sin \theta)^2}{2g}$   
 $= R + H, R = \frac{2u^2 \sin \theta \cos \theta}{g}$   
 $H = \frac{(u \sin \theta)^2}{2g}$

7. [A] Constrain  
 $-x_2 + 2x = 0$   
 $x_2 = 2x$   
 $-x_1 + x_2 + x_2$   
 $x_1 = 2x_2$   
 $x_1 = 4x$



$$2Kx_1 = \frac{\Delta T}{2}$$

$$\Delta T = 4 Kx_1$$

$$\Rightarrow 16 Kx$$

$$a = \frac{16 Kx}{m}$$

$$\omega = G \sqrt{\frac{K}{m}}$$

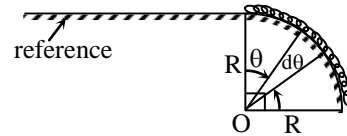
$$T = \frac{2\pi}{4} \sqrt{\frac{m}{K}} = \frac{\pi}{2} \sqrt{\frac{m}{K}}$$

8. [D]  $H = \frac{Q}{t} = \frac{KA}{\ell} (\theta_1 - \theta_2)$   
 $= \frac{mL}{t} = \frac{K4\pi r^2}{\ell} (\theta_1 - \theta_2)$   
 $\frac{V\rho L}{t} = \frac{K4\pi r^2}{\ell} (\theta_1 - \theta_2)$   
 $\frac{4\pi r^3 \rho L}{3t} = \frac{K4\pi r^2}{\ell} (\theta_1 - \theta_2)$   
 $K \propto \frac{r \cdot \ell}{t}$

$$\frac{K_1}{K_2} = \frac{r_1}{r_2} \cdot \frac{\ell_1}{\ell_2} \times \frac{t_2}{t_1}$$

$$\frac{K_1}{K_2} = \frac{2r}{r} \cdot \frac{\ell}{4\ell} \cdot \frac{16}{25} = \frac{8}{25}$$

9. [C]



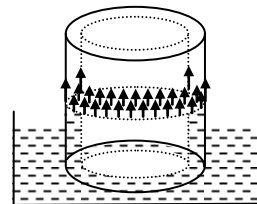
$$dU_i = - \left( \frac{m}{\ell} R d\theta \right) \times g \times R [1 - \cos \theta]$$

$$dU_i = - \frac{mgR^2}{\ell} [1 - \cos \theta] d\theta$$

$$\therefore U_i = - \frac{mgR^2}{\ell} \left[ \left( \frac{\ell}{R} \right) - \sin \left( \frac{\ell}{R} \right) \right]$$

and  $U_f = 0 \therefore W_{\text{ext}} = \Delta U$

10. [D]



Net upward force  
 $= 2\pi R_2 S + 2\pi R_1 S \quad \text{contact angle} = 0^\circ$   
 $\therefore$  Capillary rise is given by

$$h = \frac{2\pi S(R_1 + R_2)}{\pi(R_2^2 - R_1^2)\rho g}$$

$$= \frac{2S}{(R_2 - R_1)\rho g}$$

**Passage # 1 (Q.11 & 12)**

**Sol. 11. [B] & 12. [B]**

$$\text{Emf} = Bv\ell$$

$$\text{Here } q = CV = CBv\ell \quad \dots(i)$$

$$i = \frac{dq}{dt} = C\ell B \left( \frac{dv}{dt} \right) = CB\ell a \quad \dots(ii)$$

$$\text{Here } F_{\text{net}} = F - F_B = F - Bi\ell$$

$$ma = F - B(CB\ell a)\ell$$

$$a = \frac{F}{m + CB^2\ell^2} = \text{const} \quad \dots(iii)$$

$$\frac{dv}{dt} = \frac{F}{m + CB^2\ell^2}$$

$$V = \left( \frac{F}{m + CB^2\ell^2} \right) t \quad \dots(iv)$$

Equation (ii) says  $i$  is const

Equation (iv) says  $V \propto t$  hence velocity  $\uparrow$ s

Equation (i) says with increase in velocity charge increases

**Passage # 2 (Q.13 & 14)**

**13. [D]**

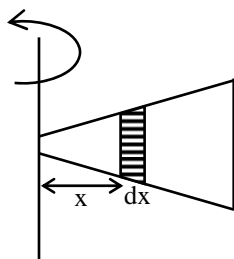


Fig.

$$\int dI = \int_0^L dm \cdot x^2$$

$$= \int_0^L \lambda dx \cdot x^2 = \int_0^L K \cdot x^3 dx = \frac{KL^4}{4}$$

So choice (D) is correct and rest are wrong.

**14. [D]** If  $M$  is mass of the square plate.

Mass of portion of each hole

$$m = \frac{M}{16R^2} \pi R^2 = \frac{\pi}{16} M$$

Moment of Inertia of remaining portion

$$I = I_{\text{square}} - 4 I_{\text{hole}}$$

$$= \frac{\pi}{16} [16R^2 + 16R^2] - 4 \left[ \frac{mR^2}{2} + m(\sqrt{2}R)^2 \right] I$$

$$= \left( \frac{8}{3} - \frac{10\pi}{16} \right) MR^2$$

**Passage # 3 (Q.15 & 16)**

**15. [A]** The force exerted by the gas on the piston at the moment when compression of the spring is  $x$ , is given by

$$F = P_0A + kx$$

where  $P_0 = 100 \text{ kPa} = \text{atmospheric pressure}$   
 $A = 20 \text{ cm}^2 = \text{Area of cross-section of the piston}$   
 and  $k = 200 \text{ N/m} = \text{spring constant}$ . Hence, work done by the gas as the piston moves through

$\ell = 10 \text{ cm}$ , is given by :

$$W = \int_0^\ell F dx = \int_0^\ell (P_0A + kx) dx = P_0A \ell + \frac{1}{2} k \ell^2$$

$$= (100 \times 10^3 \text{ N/m}^2) \times (20 \times 10^{-4} \text{ m}^2) \times (10 \times 10^{-2} \text{ m})$$

$$+ \frac{1}{2} \times (200 \text{ N/m}) \times (100 \times 10^{-4} \text{ m}^2)$$

$$= 20 \text{ J} + 1 \text{ J} = 21 \text{ J}$$

**16. [C]** The internal energy is :  $U = \left( \frac{3}{2} \right) nRT$

$$\therefore dU = \left( \frac{3}{2} \right) nR\Delta T$$

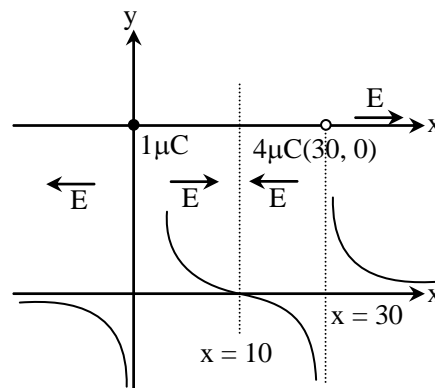
$$= 1.5 \times (2.0 \text{ mol}) \times (8.3 \text{ J/mol-K}) \times (31 \text{ K}) = 772 \text{ J}$$

Hence, according to first law of thermodynamics,

$$\Delta Q + \Delta U + \Delta W = 772 \text{ J} + 21 \text{ J} = 793 \text{ J}$$

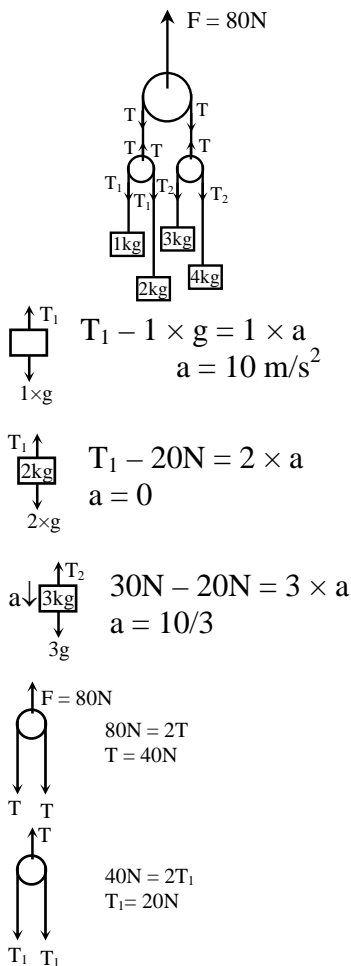
**COLUMN MATCHING :**

**17. [A]**



Electric potential decreases along electric field vector.

18. [B]



19. [C]  $S = \frac{1}{2} \times 2 \times 16 = 16 \text{ m}$

$|W_g| = mg S =$

$W_N = m(g + a) \cos^2 \theta. S$

$W_f = m(g + a) \sin^2 \theta. S$

20. [C] (1)  $10 \times 1(\theta - 30) = 10 \times 1(70 - \theta) \Rightarrow 50^\circ\text{C} \rightarrow S$

(2)  $10 \times 80 + 10 \times 1 \times (\theta - 0) = 10 \times 1 \times (85 - \theta)$   
 $\Rightarrow 2\theta = 5^\circ\text{C} \Rightarrow \theta = 2.5^\circ\text{C} \rightarrow P$

(3)  $\theta < 80^\circ\text{C}$ , so ice will not melt completely  $\rightarrow Q$

(4)  $m_s > \frac{m_i}{3}$ , so steam will not condense completely  $\rightarrow R$

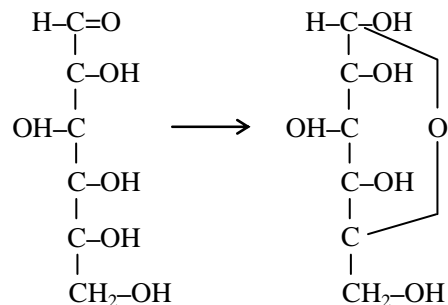
## CHEMISTRY

### Section - I

1.[D] Phenolic structure [Stabilised by Resonance]

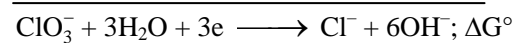
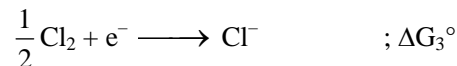
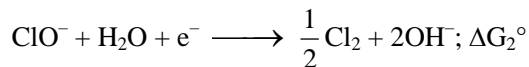
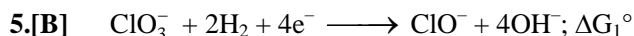
2.[D] For benzyne mechanism at least 1  $\beta$ -hydrogen is necessary.

3.[C]



(Five hydroxyl groups) (Five hydroxyl groups)

4.[C] It is given by 1° amide.



$\therefore \Delta G^\circ = \Delta G_1^\circ + \Delta G_2^\circ + \Delta G_3^\circ$

$-6FE^\circ = -4F \times 0.54 - 1F \times 0.45 - 1F \times 1.07$

$\therefore E^\circ = + \frac{3.68}{6} = +0.61 \text{ V}$

6.[D]  $n_{\text{H}^+} = 10^{-2} \text{ V}$

$n_{\text{OH}^-} = 10^{-2} \text{ V}$

so, solution is neutral & pH = 6

7.[C] The loss in weight should be proportional to vapour pressure above that solution:

So,  $P_{S_A} \propto 2\text{gm}$

$P_{S_B} \propto 1.5\text{gm}$

$P_{S_C} \propto 2.5\text{gm}$

So, maximum vapour pressure is above C solution hence, it is having minimum lower and hence minimum mole fraction (hence minimum number of moles of solute) So max, molar mass of substance.

8.[B]  $r_2 = k_2 [A]_2^1 [B]_2^1$  for a certain run

$r_1 = k_1 [A]_1^1 [B]_1^1$  for a previous run

$\frac{r_2}{r_1} = \frac{k_2 [A]_2 [B]_2}{k_1 [A]_1 [B]_1}$

Substituting the given information

$1.5 = 2^{\left(\frac{t_2-27}{10}\right)} \times \frac{1}{2} \times \frac{1}{2}$

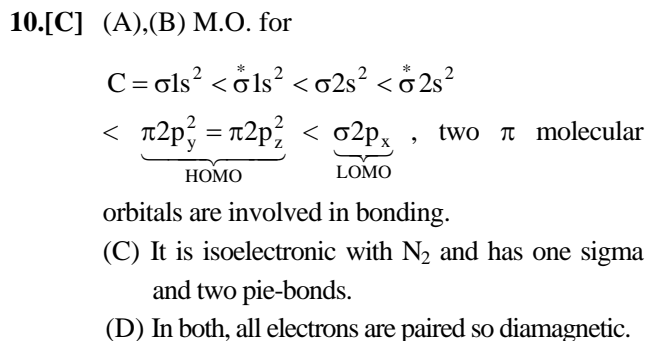
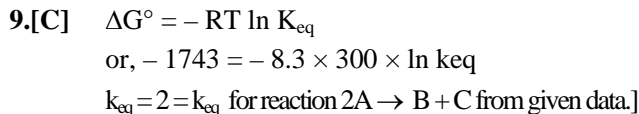
$\Rightarrow 6 = 2^{\frac{t_2-27}{10}}$

$\Rightarrow \frac{t_2-27}{10} \ln 2 = \ln 6$

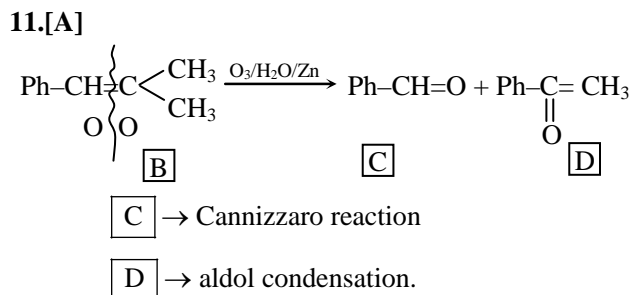
$$\Rightarrow \frac{t_2 - 27}{10} = \frac{\ln 6}{\ln 2}$$

$$\Rightarrow \frac{t_2 - 27}{10} = 2.585$$

$$\Rightarrow t_2 = 52.85^\circ\text{C} = 53^\circ\text{C}$$



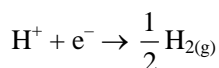
**Passage # 1 (Ques. 11 & 12)**



**Passage # 2 (Ques. 13 & 14)**

$$n_{e^-} = \frac{i \times t}{96500} = \frac{1.25 \times 212 \times 60}{96500} = 0.165$$

13.[A] At anode



$$n_{e^-} = n_{H^+} = 0.165$$

14.[C] At cathode

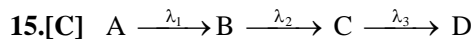
$$n_{e^-} = n_{OH^-} = 0.165$$



$t = 0$                       0.33                      0.165                      0.33  
 (after reaction) (0.33 - 0.165)    0    (0.33 + 0.165)

therefore  $pH = 7.2 + \log_{10} \frac{0.469}{0.165} = 7.65$

**Passage # 3 (Ques. 15 & 16)**



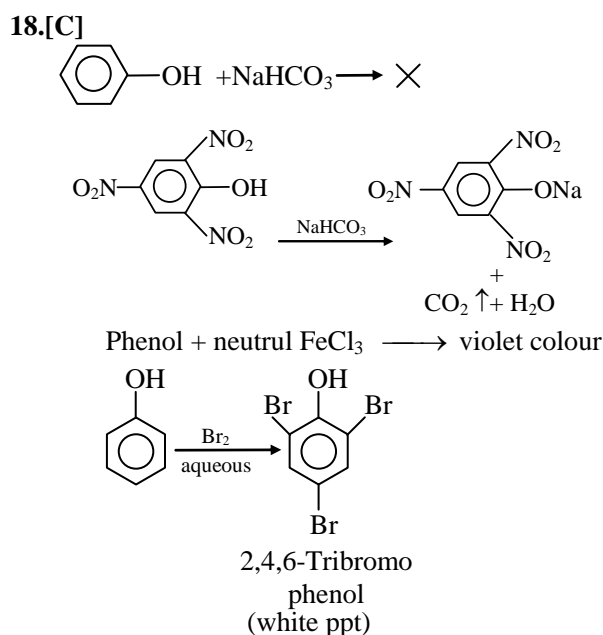
Since  $\lambda_1 \gg \lambda_2 \ll \lambda_3$

We can assume that all the 'A' has been converted into 'B' in small duration  
 Number of moles of C formed = number of moles of 'B' dissociated

$$\Delta B = \lambda_2 N t = \frac{\ln 2}{6930} \times 10^{20} \times 10 = 10^{17}$$

16.[B] Number of nuclei of 'D' formed = number of nuclei of 'B' disintegrated =  $\frac{1}{2} \times 10^{20}$

**Section - III**



19.[B] Theory

20.[D] Theory

**MATHEMATICS**

**Section - I**

1.[B] Since equation  $5x^2 + 12x + 13 = 0$  has imaginary roots.

Hence equations  $ax^2 + bx + c = 0$  &

$5x^2 + 12x + 13 = 0$  have both common roots

$$\therefore \frac{a}{5} = \frac{b}{12} = \frac{c}{13} = k \text{ (say)}$$

$$a = 5k, b = 12k, c = 13k$$

$$\Rightarrow a^2 + b^2 = 25k^2 + 144k^2 = 169k^2 = (13k)^2 = c^2$$

$$\Rightarrow a^2 + b^2 = c^2$$

$\Rightarrow$  ABC is right angled triangle

2.[C] For the given system to have a non-trivial solution, we must have

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & k & 3 \\ 0 & -2k & -11 \\ 0 & 3-2k & -10 \end{vmatrix} = 0$$

[Applying  $R_2 \rightarrow R_2 - 3R_1, R_3 \rightarrow R_3 - 2R_1$ ]

$$\Rightarrow 20k + 11(3 - 2k) = 0$$

$$\Rightarrow k = \frac{33}{2}$$

3.[D]  $\left| \frac{z-i}{z+i} \right| = \text{purely real}$

$$\arg \left| \frac{z-i}{z+i} \right| = 0$$

$$\int_2^4 [0] dx = 0$$

4.[A]  $S_1 : \sin^{-1}x - \frac{\pi}{2} + \sin^{-1}(-x) = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}(x) + \sin^{-1}(-x) = \pi$$

$0 = \pi$  which is not possible

$\therefore$  no solution

$$S_2 : \sin^{-1}(x^2 + 4x + 3) + \cos^{-1}(x^2 + 6x + 8) = \frac{\pi}{2}$$

$$= \sin^{-1}(x^2 + 4x + 3) + \cos^{-1}(x^2 + 4x + 3)$$

$$\Rightarrow x^2 + 6x + 8 = x^2 + 4x + 3$$

$$\Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$$

$$\therefore x^2 + 4x + 3 = (x+2)^2 - 1 \in [-1, 1]$$

$$\text{at } x = -\frac{5}{2}$$

$$\& x^2 + 6x + 8 = (x+3)^2 - 1 \in [-1, 1]$$

$$\text{at } x = -\frac{5}{2}$$

$$\therefore x = -\frac{5}{2}$$

$$S_3 : \because -1 \leq \cos(\sin^{-1}x) \leq 1$$

$$\text{and } -1 \leq \sin(\cos^{-1}x) \leq 1$$

$$\sin^{-1}\{\cos(\sin^{-1}x)\} + \cos^{-1}\{\sin(\cos^{-1}x)\} = \frac{\pi}{2}$$

$$S_4 : 2 \left[ \tan^{-1} \frac{1+2}{1-2} + \pi + \tan^{-1} 3 \right]$$

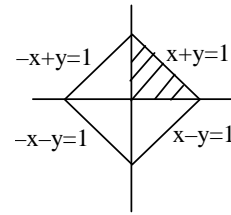
$$= 2[\pi - \tan^{-1}3 + \tan^{-1}3] = 2\pi$$

5.[A]  $S_1 : a_{ji} = j^2 - i^2 = -(i^2 - j^2)$

$$a_{ji} = -a_{ij}$$

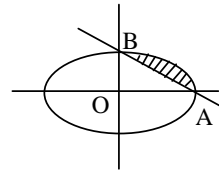
Skew-symmetric matrix

$$S_2 : \text{Area} = 4 \left( \frac{1}{2} \cdot 1 \cdot 1 \right) = 2$$

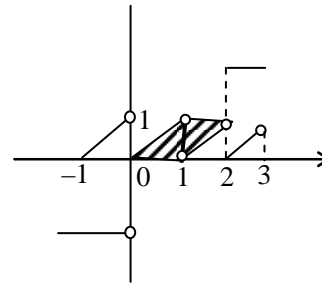


$S_3 : \text{Area} = \frac{1}{4} (\text{Ellipse area}) - \Delta OAB$

$$= \frac{\pi ab}{4} - \frac{ab}{2}$$



$S_4 : \text{Area} = 2 \cdot \left( \frac{1}{2} \cdot 1 \cdot 1 \right) = 1$



6.[A]  $S_1 : [\sin^{-1} x] = \{1 + x^2\} = \{x^2\}$

$$[\sin^{-1} x] = \begin{cases} -2 & , -1 \leq x < -\sin 1 \\ -1 & , -\sin 1 \leq x < 0 \\ 0 & , 0 \leq x < \sin 1 \\ 1 & , \sin 1 \leq x < 1 \end{cases}$$

$\therefore$  the only solution is  $x = 0$

$S_2 : \text{Range of } f(x) \text{ is whole of } \mathbb{R}$

$S_2$  is true

$S_3 : \text{No basic inverse trigonometric function is periodic.}$

$$S_4 : (x^2 - 3x - 10) \ln^2(x - 3) \geq 0$$

$$\text{i.e. } x^2 - 3x - 10 \geq 0, x = 4$$

$$\text{i.e. } x \in [5, \infty) \cup \{4\}$$

$\therefore S_4$  is false

7.[A]  $S_1 : x^2 + 4y^2 - 2x - 16y + 13 = 0$

$$\text{i.e. } (x^2 - 2x + 1) + 4(y^2 - 4y + 4) = 4$$



$$\text{i.e. } \frac{(x-1)^2}{4} + \frac{(y-2)^2}{1} = 1$$

$$\therefore \text{ length of latus rectum} = \frac{2 \times 1}{2} = 1$$

$\therefore S_1$  is true

**S<sub>2</sub>**: For the ellipse  $a = 2, b = 1$

$$\therefore e = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore 2ae = 2.2. \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$\therefore S_2$  is false.

**S<sub>3</sub>**: Sum of the focal distance =  $2a = 4$

$\therefore S_3$  is true

**S<sub>4</sub>**: tangents at the vertices are

$$x - 1 = \pm 2$$

$$x = 3, -1$$

$\therefore$  the line  $y = 3$  intersect there as

P (3, 3) and Q (-1, 3)

A focus is  $x = 1 + \sqrt{3}, y = 2$

i.e. focus is S ( $\sqrt{3} + 1, 2$ )

$$\text{slope of PS is } \frac{1}{2-\sqrt{3}}, \text{ slope of QS is } \frac{1}{-2-\sqrt{3}}$$

$$\therefore \text{ Product of slopes} = \frac{1}{2-\sqrt{3}} \times \frac{1}{-2-\sqrt{3}} = -1$$

$\therefore S_4$  is true

$$8.[D] \quad f(x) = \frac{x^2 + 5x - 14}{x^2 - 7x + 10} = \frac{x+7}{x-5}$$

$$\therefore f(2) = \frac{-9}{3} = -3$$

$$9.[B] \quad p = \lim_{n \rightarrow \infty} \left[ \frac{\prod_{r=1}^n (n^3 + r^3)}{n^{3n}} \right]^{1/n}$$

$$\ln p = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \ln \left( 1 + \left( \frac{r}{n} \right)^3 \right)$$

$$= \int_0^1 \ln(1+x^3) dx = \ln 2 - 3 + 3\lambda$$

10.[B] Given,

$$1 \geq |z - (4 - 3i)| \geq \begin{cases} |z| - |4 - 3i| \\ |4 - 3i| - |z| \end{cases}$$

$$\Rightarrow |z| \leq 6 \text{ and } |z| \geq 4$$

$$\Rightarrow 4 \leq |z| \leq 6 \Rightarrow \alpha = 4, \beta = 6$$

$$\text{Let } y = \frac{x^4 + x^2 + 4}{x} = x^3 + x + \frac{4}{x}$$

$$= x^3 + x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x}$$

Since  $x \in (0, \infty)$ , therefore

$$x^3, x, \frac{1}{x} \text{ are positive.}$$

Sum will be least when  $x^2 = x = \frac{1}{x}$

$$\Rightarrow x = 1$$

$$\therefore k = 6$$

Hence,  $k = \beta$

### Passage # 1

#### 11.[C] & 12.[D]

$$f(xy) = xf(y) + yf(x) \quad \forall x, y \in \mathbb{R}$$

replace  $x$  by 1,  $y$  by  $x$

$$f(x) = f(x) + xf(1) \Rightarrow f(1) = 0$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f\left(x + \frac{h}{x}\right) - f(x)}{\frac{h}{x}}$$

$$= \lim_{h \rightarrow 0} \frac{xf\left(1 + \frac{h}{x}\right) + \left(1 + \frac{h}{x}\right)f(x) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f\left(1 + \frac{h}{x}\right) - f(1)}{\frac{h}{x}} + \frac{f(x)}{x}$$

$$f'(x) = f'(1) + \frac{f(x)}{x}$$

$$f'(x) - \frac{f(x)}{x} = f'(1) = 1$$

$$\frac{f'(x)}{x} - \frac{f(x)}{x^2} = \frac{1}{x}$$

$$\int d\left(\frac{f(x)}{x}\right) = \int \frac{1}{x} dx$$

$$\frac{f(x)}{x} = \ln x + c$$

$$f(x) = x \ln x + cx$$

$$f(1) = 0 = 0 + c \Rightarrow c = 0$$

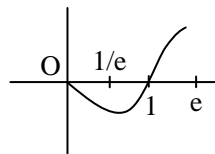
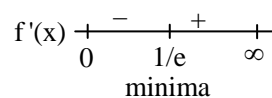
$$f(x) = x \ln x$$

$$\lim_{x \rightarrow 0^+} (1 + f(x))^{1/x} = \lim_{x \rightarrow 0^+} e^{\frac{f(x)}{x}}$$

$$= e^{\lim_{x \rightarrow 0^+} \ln x} = e^{-\infty} = 0$$

$$f(x) = x \ln x$$

$$f'(x) = \ln x + x \cdot \frac{1}{x} = 1 + \ln x$$



### Passage # 2

13.[B] & 14.[A]

Since no point of the parabola is below x-axis

$$\therefore a^2 - 4 \leq 0$$

$\therefore$  maximum value of a is 2

Equation of the parabola, when a = 2 is

$$y = x^2 + 2x + 1$$

it intersects y-axis at (0, 1)

equation of the tangent at (0, 1) is  $y = 2x + 1$

since  $y = 2x + 1$  touches the circle  $x^2 + y^2 = r^2$

$$\therefore r = \frac{1}{\sqrt{5}}$$

Equation of the tangent at (0, 1) to the parabola

$$y = x^2 + ax + 1 \text{ is } \frac{y+1}{2} = \frac{a}{2}(x+0) + 1$$

$$\text{i.e. } ax - y + 1 = 0$$

$$\therefore r = \frac{1}{\sqrt{a^2 + 1}}$$

radius maximum when a = 0

$\therefore$  equation of the tangent is  $y = 1$

$\therefore$  slope of the tangent is 0

### Passage # 3

15.[A] & 16.[C]

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1}$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x+1} = x+1 - \frac{3}{x+1}$$

$$\Rightarrow d\left(\frac{y}{x+1}\right) = \int 1 - \frac{3}{(x+1)^2} dx$$

$$\frac{y}{x+1} = x + \frac{3}{x+1} + c$$

$$y = (x+c)(x+1) + 3$$

(2, 0) lies on it

$$0 = (2+c)(2+1) + 3$$

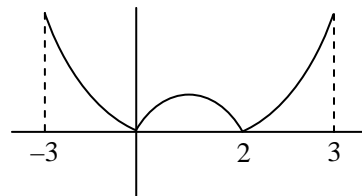
$$2+c+1=0 \Rightarrow c=-3$$

$$y = (x-3)(x+1) + 3$$

$$y = x^2 + x - 3x \Rightarrow y = x^2 - 2x$$

Parabola

$$y = |x^2 - 2x|$$



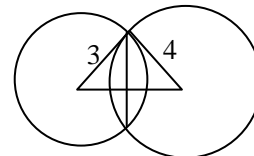
$$\text{Area} = \int_{-3}^0 (x^2 - 2x) dx + \int_0^2 (2x - x^2) dx$$

$$+ \int_2^3 (x^2 - 2x) dx$$

$$= \left(\frac{x^3}{3} - x^2\right) \Big|_{-3}^0 + \left(x^2 - \frac{x^3}{3}\right) \Big|_0^2 + \left(\frac{x^3}{3} - x^2\right) \Big|_2^3$$

$$= \frac{62}{3}$$

17.[C] (P) Let length of common chord be 2a, then



$$\sqrt{9 - a^2} + \sqrt{16 - a^2} = 5$$

$$\sqrt{16 - a^2} = 5 - \sqrt{9 - a^2}$$

$$16 - a^2 = 25 + 9 - a^2 - 10\sqrt{9 - a^2}$$

$$10\sqrt{9 - a^2} = 18 \Rightarrow 100(9 - a^2) = 324$$

$$\text{i.e. } 100a^2 = 576$$

$$\therefore a = \sqrt{\frac{576}{100}} = \frac{24}{10}$$

$$\therefore 2a = \frac{24}{5} = \frac{k}{5} \Rightarrow k = 24$$

(Q) Equation of common chord is

$$6x + 4y + p + q = 0$$

Common chord pass through centre (-2, -6) of

$$\text{circle } x^2 + y^2 + 4x + 12y + p = 0$$

$$\therefore p + q = 36$$

(R) Equation of the circle is  $2x^2 + 2y^2 - 2\sqrt{2}x - y = 0$

Let  $(\alpha, 0)$  be mid point of a chord. Then equation of the chord is

$$2\alpha x - \sqrt{2}(x + \alpha) - \frac{1}{2}(y + 0) = 2\alpha^2 - 2\sqrt{2}\alpha$$

Since it passes through the point  $(\sqrt{2}, \frac{1}{2})$

$$\therefore 2\sqrt{2}\alpha - \sqrt{2}(\sqrt{2} + \alpha) - \frac{1}{4} = 2\alpha^2 - 2\sqrt{2}\alpha$$

$$\text{i.e. } 8\alpha^2 - 12\sqrt{2}\alpha + 9 = 0$$

$$\text{i.e. } (2\sqrt{2}\alpha - 3)^2 = 0$$

$$\text{i.e. } \alpha = \frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}} \quad \therefore \text{ number of chords is } 1$$

(S) Mid point of AB = (1, 4)

$\therefore$  Equation of perpendicular bisector of AB is  $x = 1$

A diameter is  $4y = x + 7$

$\therefore$  Centre of the circle is (1, 2)

$\therefore$  sides of the rectangle are 8 and 4

$\therefore$  area = 32

18.[A] (P)  $f(x) = |\sin x| + |\cos x|$

Period is  $\frac{\pi}{2}$ .

$$(Q) \vec{P} = ((\vec{a} \times \vec{b}) \cdot \vec{c})\vec{b} - ((\vec{a} \times \vec{b}) \cdot \vec{b})\vec{c} = [\vec{a}\vec{b}\vec{c}]\vec{b}$$

Similarly  $\vec{q} = [\vec{a}\vec{b}\vec{c}]\vec{c}$  and  $\vec{r} = [\vec{a}\vec{b}\vec{c}]\vec{a}$

$$\Rightarrow [\vec{p}\vec{q}\vec{r}] = [\vec{a}\vec{b}\vec{c}]^4 \Rightarrow n = 4$$

(R) as  $\sqrt{\frac{a^2 + 2b^2}{a}} > b$ , points must lie on major axis

$$\text{Now } a = \sqrt{\frac{a^2 + 2b^2}{2}} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$$

$$\therefore e = \frac{1}{\sqrt{2}} \Rightarrow k = 2$$

(S) For  $\Delta ABC$

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

$$\text{Also } 2\cot B = \cot A + \cot C$$

A.M.  $\geq$  G.M.

$$\frac{\cot A + \cot C}{2} \geq \sqrt{\cot A \cot C}$$

$$\Rightarrow \cot^2 B \geq \cot A \cot C$$

$$\geq 1 - \cot B (\cot A + \cot C)$$

$$\geq 1 - 2\cot^2 B$$

$$\Rightarrow \cot^2 B \geq \frac{1}{3}$$

$$\Rightarrow (\cot B)_{\min} = \frac{1}{\sqrt{3}}$$

$$\therefore k = 3$$

$$19.[D] (P) S = \lim_{n \rightarrow \infty} \sum_{n=1}^n \frac{{}^n C_2}{2^n}$$

$$S = \frac{1}{4} + \frac{3}{8} + \frac{6}{16} + \frac{10}{32} + \dots \quad \dots(1)$$

$$\frac{S}{2} = \frac{1}{8} + \frac{3}{16} + \frac{6}{32} + \dots \quad \dots(2)$$

$$\frac{S}{2} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots \quad \dots(3)$$

[from (1) - (2)]

$$\frac{S}{4} = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \dots \quad \dots(4)$$

$$\frac{S}{4} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \quad \text{[From (3) - (4)]}$$

$$\Rightarrow \frac{S}{4} = \frac{(1/4)}{1 - (1/2)} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} \Rightarrow S = 2$$

(Q) Uncommon roots when  $g(x) \neq 0$  and  $f(x)$  is zero are 2, 9 which are the solutions

$$(R) y = \frac{1 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} - 2\sin x \cos x$$

$$= (\tan x + \cot x) - \sin 2x$$

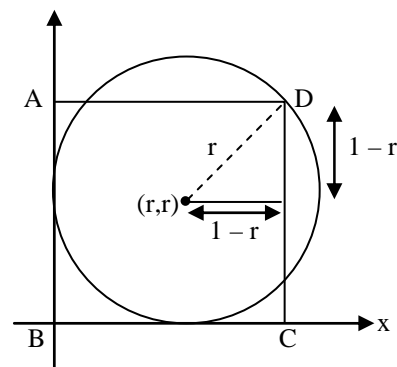
Now,  $(\tan x + \cot x)$  is minimum at  $x = \pi/4$  and  $\sin 2x$  is maximum at  $x = \pi/4$

$\therefore y_{\min}$  occurs at  $x = \pi/4$  and  $y_{\min} = 1$

$$(S) r^2 = 2(1 - r)^2$$

$$r^2 - 4r + 2 = 0$$

$$r = 2 - \sqrt{2}$$



20.[B] (P) Put  $x = 2$  in both equation we get

$$4 + 2a + b = 0 \quad \dots\dots(i)$$

$$4 + 2c + d = 0 \quad \dots\dots(ii)$$

(i) - (ii)

$$\Rightarrow \frac{(b-d)}{(c-a)} = 2$$

(Q) No. of arrangements =  $\frac{6!}{2!} = 3.5!$

(R) Last non zero digit in  $21!$  can be obtained by using exponent method

$$\text{No. of 2's} = \left[ \frac{21}{2} \right] + \left[ \frac{21}{4} \right] + \left[ \frac{21}{8} \right] + \left[ \frac{21}{16} \right] = 18$$

$$\text{No. of 5's} = \left[ \frac{21}{5} \right] = 4$$

$\therefore$  No. of zeroes = 4

$$\text{No. of 3's} = \left[ \frac{21}{3} \right] + \left[ \frac{21}{9} \right] = 9$$

$\therefore$  Last non zero digit will be obtained

$$\text{from } 2^{14} \times 3^9 \times 7^3 \times 11 \times 13 \times 15 \times 17 \times 19 = 4$$

(S)  $\therefore 37^{n+2} + 16^{n+1} + 30^n$   
 $= (35 + 2)^{n+2} + (14 + 2)^{n+1} + (28 + 2)^n$   
 $= 7k + 2^{n+2} + 2^{n+1} + 2^n$   
 $= 7k + 2^n \cdot 7$

$\therefore$  remainder = 0