Part – I (CHEMISTRY)

SECTION – I

CODE - 1

Straight Objective Type

11/04/2010

This section contains 8 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. The bond energy (in kcal mol^{-1}) of a C–C single bond is approximately

(A) 1 (B) 10 (C) 100 (D) 1000

Ans. **[C]**

Sol. Value is 82.6 kcal/mol.

2. The species which by definition has **ZERO** standard molar enthalpy of formation at 298 K is

(A) $Br_2(g)$ (B) Cl_2 (g) (C) H_2O (g) (D) CH_4 (g)

Ans. **[B]**

Sol. Because standard state of Cl_2 is gas.

3. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is

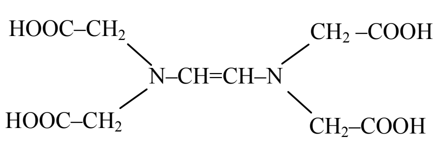
(A) $[Cr(H_2O)_4(O_2N)]Cl_2$ (B) $[Cr(H_2O)_4Cl_2](NO_2)$

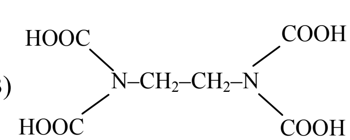
(C) $[Cr(H_2O)_4Cl(ONO)]Cl$ (D) $[Cr(H_2O)_4Cl_2(NO_2)].H_2O$

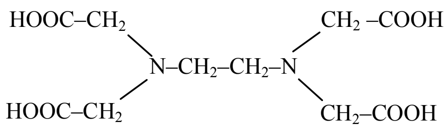
Ans. **[B]**

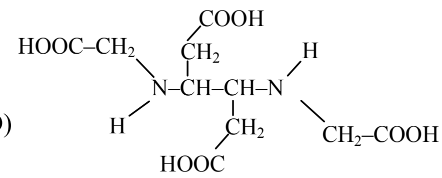
Sol. Ionization isomer have different ions in solution

4. The correct structure of ethylenediaminetetraacetic acid (EDTA) is -

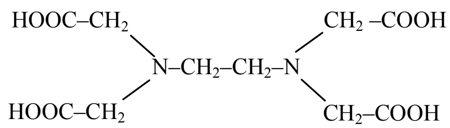
(A) 

(B) 

(C) 

(D) 

Ans. [C]

Sol. Structure is 

5. The synthesis of 3-octyne is achieved by adding a bromoalkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are

(A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$ (B) $BrCH_2CH_2CH_3$ and $CH_3CH_2\ CH_2\ C\equiv CH$

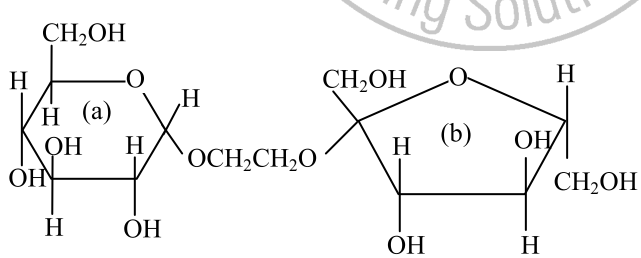
(C) $BrCH_2CH_2CH_2\ CH_2CH_3$ and $CH_3C\equiv CH$ (D) $BrCH_2CH_2CH_2CH_3$ and $CH_3C_2\ C\equiv CH$

Ans. [D]

Sol. (i) $CH_3–CH_2–C\equiv CH \xrightarrow{NaNH_2} CH_3CH_2–C\equiv \overset{-}{C}\,Na^+$

(ii) $CH_3–CH_2–CH_2–CH_2–Br + CH_3CH_2–C\equiv \overset{-}{C}\,Na^+ \longrightarrow CH_3–CH_2–CH_2–CH_2–C\equiv C–CH_2–CH_3$

6. The correct statement about the following disaccharide is



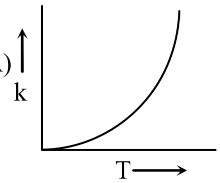
(A) Ring (a) is pyranose with α-glycosidic link (B) Ring (a) is furanose with α-glycosidic link

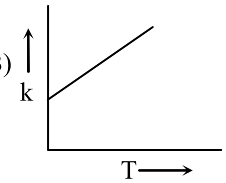
(C) Ring (b) is furanose with α-glycosidic link (D) Ring (b) is Pyranose with β-glycosidic link

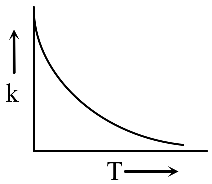
Ans. [A]

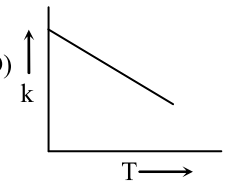
Sol. Ring (a) is pyranose with α-glycosidic link

7. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is -

(A) 

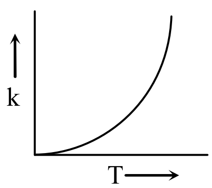
(B) 

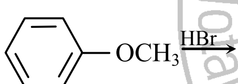
(C) 

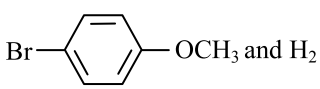
(D) 

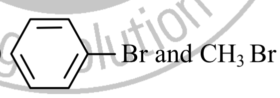
Ans. **[A]**

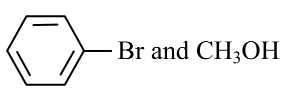
Sol. $k = Ae^{-Ea/RT}$; $k \propto e^{-E_a/RT}$

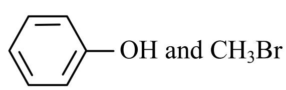


8. In the reaction  the products are

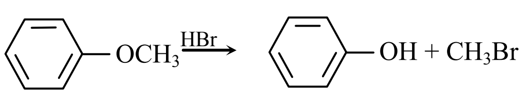
(A) 

(B) 

(C) 

(D) 

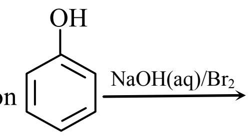
Ans. **[D]**

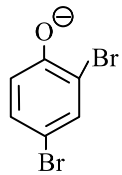
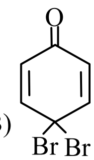
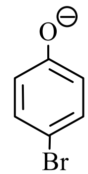
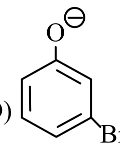
Sol. 

SECTION – II

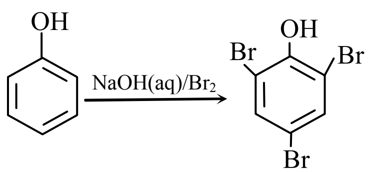
Multiple Correct Answers Type

This section contains 5 multiple choice questions. Each questions has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

9. In the reaction  the intermediate(s) is (are)

(A)  (B)  (C)  (D) 

Ans. **[A, B, C]**

Sol. 

Bromination take place at ortho & para position due to activation of benzene ring by –OH group.

10. The reagent(s) used for softening the temporary hardness of water is (are) -

(A) $Ca_3(PO_4)_2$ (B) $Ca(OH)_2$ (C) Na_2CO_3 (D) NaOCl

Ans. **[B, C, D]**

11. Aqueous solutions of HNO_3, KOH, CH_3COOH and CH_3COONa of identical concentrations are provided. The pair(s) of solutions which from a buffer upon mixing is(are)

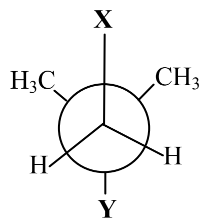
(A) HNO_3 and CH_3COOH (B) KOH and CH_3COONa

(C) HNO_3 and CH_3COONa (D) CH_3COOH and CH_3COONa

Ans. **[C, D]**

Sol. Acidic buffer is made up of weak acid & it's conjugate ion

12. In the Newman projection for 2,2-dimethylbutane

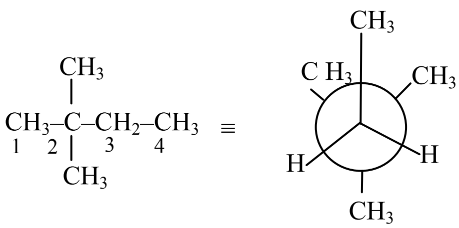


X and Y can respectively be

(A) H and H (B) H and C_2H_5 (C) C_2H_5 and H (D) CH_3 and CH_3

Ans. **[B, D]**

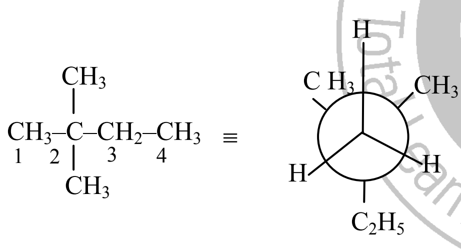
Sol.



along C_2–C_3 Bond **Option [D]**

X = CH_3

Y = CH_3



along C_1–C_2 **Option [B]**

X = H

Y = C_2H_5

13. Among the following, the intensive property is (properties are)

(A) molar conductivity (B) electromotive force

(C) resistance (D) heat capacity

Ans. **[A, B]**

SECTION – III
Paragraph Type

This section contains **2 paragraphs**. Based upon the first paragraph **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which O**NLY ONE** is correct.

Paragraph for questions 14 to 15

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is :

M(s) | M^+ (aq; 0.05 mloar) || M^+ (aq; 1 molar) | M(s)

For the above electrolytic cell the magnitude of the potential $|E_{cell}|$ = 70 mV.

14. For the above cell

(A) $E_{cell} < 0; \Delta G > 0$ (B) $E_{cell} > 0; \Delta G < 0$

(C) $E_{cell} < 0; \Delta G^o > 0$ (D) $E_{cell} > 0; \Delta G^o < 0$

Ans. **[B]**

Sol. For concentration cell

$E_{cell} = -\frac{k}{n}\log\frac{0.05}{1}$ [where k is +ve constant]

$= +$ ve

$\therefore E_{cell} > 0 ; \Delta G < 0$

15. If the 0.05 molar solution of M^+ is replaced by a 0.0025 molar M^+ solution, then the magnitude of the cell potential would be

(A) 35 mV (B) 70 mV (C) 140 mV (D) 700 mV

Ans. **[C]**

Sol. $E_{cell} = -\frac{k}{n}\log\frac{0.0025}{1}$

$$= 2\left[-\frac{k}{n}\log\frac{0.05}{1}\right] = 2 \times 70 = 140$$

Paragraph for Questions 16 to 18

Copper is the most noble of the first row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite ($CuSO_4$. $5H_2O$), atacamite ($Cu_2Cl\ (OH)_3$), cuprite (Cu_2O), copper glance (Cu_2S) and malachite ($Cu_2(OH)_2CO_3$). However, 18% of the world copper production come from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self-reduction.

16. Partial roasting of chalcopyrite produces

(A) Cu_2S and FeO (B) Cu_2O and FeO

(C) CuS and Fe_2O_3 (D) Cu_2O and Fe_2O_3

Ans. **[A]**

17. Iron is removed from chalcopyrite as -

(A) FeO (B) FeS (C) Fe_2O_3 (D) $FeSiO_3$

Ans. **[D]**

18. In self-reduction, the reducing species is -

(A) S (B) O^{2-} (C) S^{2-} (D) SO_2

Ans. **[C]**

SECTION – IV

Integer Type

This section contains. **TEN** questions. The answer to each questions is a **single digit integer** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. The number of neutrons emitted when $^{235}_{92}U$ undergoes controlled nuclear fission to $^{142}_{54}Xe$ and $^{90}_{38}Sr$ is

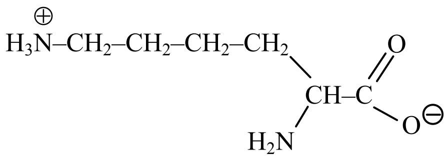
Ans. **[4]**

Sol. $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{142}_{54}Xe + ^{90}_{38}Sr + x\,^{1}_{0}n + y\,^{1}_{1}p$

235 + 1 = 142 + 90 + x

x = 4

20. The total number of basic groups in the following form of lysine is

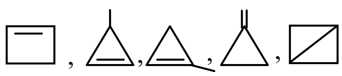


Ans. **[2]**

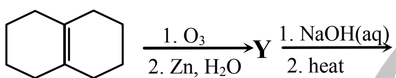
21. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is

Ans. **[5]**

Sol.

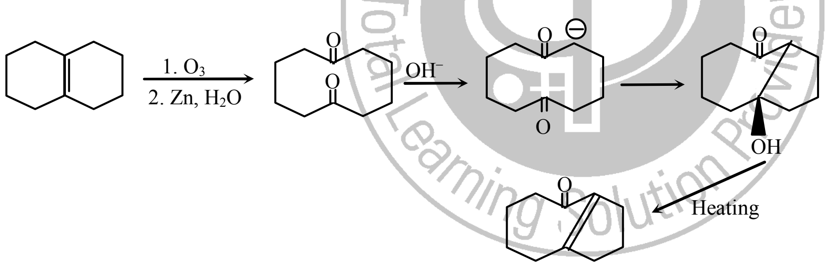


22. In the scheme given below, the total number of intramolecular aldol condensation products form '**Y**' is



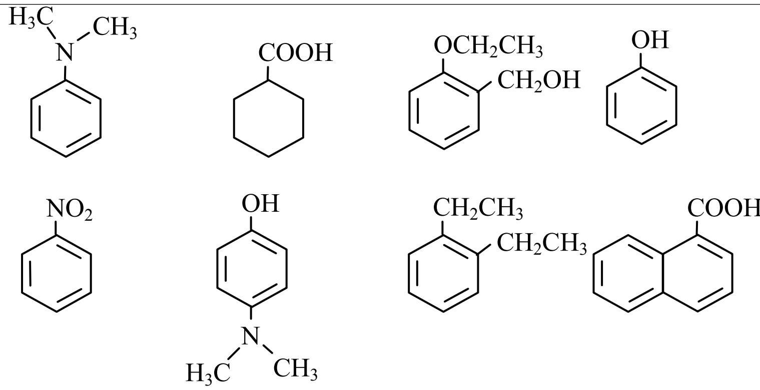
Ans. **[1]**

Sol.



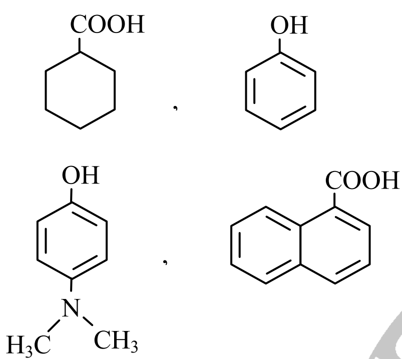
(Only one product)

23. Amongst the following, the total number of compound soluble in aqueous NaOH is



Ans. **[4]**

Sol.



24. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is

KCN K_2SO_4 $(NH_4)_2C_2O_4$ NaCl $Zn(NO_3)_2$

$FeCl_3$ K_2CO_3 NH_4NO_3 LiCN

Ans. **[3]**

Sol. Basic salt are KCN, LiCN, K_2CO_3

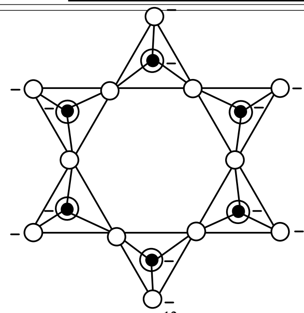
25. Based on VSEPR theory, the number of 90 degree F–Br–F angles in BrF_5 is

Ans. **[0]**

26. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is

Ans. **[3]**

Sol. $Be_3Al_2Si_6O_{18}$



$Si_6O_{18}^{-12}$ ion

27. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL and 25.0 mL. The number of significant figures in the average titre value is

Ans. **[3]**

Sol. $\frac{25.2+25.25+25.0}{3} = 25.15$

Significant figure = 3

Significant figure in the answer can not be more than least significant figure any given value.

28. The concentration of R in the reaction R $\rightarrow$ P was measured as a function of time and the following data is obtained :

[R] (molar)	1.0	0.75	0.40	0.10
t (min.)	0.0	0.05	0.12	0.18

The order of the reaction is

Ans. **[0]**

Sol. R $\rightarrow$ P

Assume zero order

$R = [R_0] - kt$

$k = \frac{[R_0]-[R]}{t}$

$k_1 = \frac{1-0.75}{0.5} = 5$

$k_2 = \frac{1-0.4}{0.12} = 5$

$k_3 = \frac{1-0.1}{0.18} = 5$

$\therefore$ order of reaction should be zero.

Part – II : (MATHEMATICS)

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

29. Let f , g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a , b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) a = b and c $\neq$ b (B) a = c and a $\neq$ b

(C) a $\neq$ b and c $\neq$ b (D) a = b = c

Ans.[D]

Sol. $f'(x) = 2x(\left(e^{x^2} - e^{-x^2}\right)$

$g'(x) = e^{x^2}\left(2x^2 - 2x + 1\right)$

$h'(x) = 2x^3e^{x^2}$

$\because$ all f'(x), g'(x), h'(x) are positive so all attains absolute maxima at x = 1

So $\because$ $f(1) = g(1) = h(1) = e + e^{-1} = a = b = c$

30. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is -

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Ans.[B]

Sol. $\alpha + \beta = -p$ ……….(1)

$\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = q$

$\Rightarrow (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta) = q$

$\Rightarrow (-p)(p^2 - 3\alpha\beta) = q$

$\alpha\beta = \frac{q + p^3}{3p}$ …..(2)

Now $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

(Sum of root) $S = \frac{p^3 - 2q}{p^3 + q}$ using (1) and (2)

(Product of root) $P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

31. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) $x + 2y - 2z = 0$

(B) $3x + 2y - 2z = 0$

(C) $x - 2y + z = 0$

(D) $5x + 2y - 4z = 0$

Ans.[C]

Sol. Plane passing through origin (0, 0, 0) and normal vector to plane is perpendicular to $3\hat{i}+4\hat{j}+2\hat{k}$, $4\hat{i}+2\hat{j}+3\hat{k}$ and $2\hat{i}+3\hat{j}+4\hat{k}$ i.e. normal vector to plane is $\hat{i}-2\hat{j}+\hat{k}$ so equation to plane is $x - 2y + z = 0$.

32. If the angle A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is -

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Ans.[D]

Sol. $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$

$\frac{a}{c}2\sin C\ \cos C+\frac{c}{a}2\sin A\cos A$

$=\frac{a\cos C+c\cos A}{R}=\frac{b}{R}=\frac{2R\sin B}{R}=\sqrt{3}$

33. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Ans.[C]

Sol. $n(s) = 6^3$

r_1	r_2	r_3
1	1	x
	2	3,6
	3	2,5
	4	x
	5	3,6
	6	2,5
2	1	3,6
	2	x
	3	1,4
	4	3,6
	5	x
	6	1,4
3	1	2,5
	2	1,4
	3	x
	4	2,5
	5	1,4
	6	x

Similarly total number of elements in events set is 48

$$= \frac{48}{216} = \frac{12}{54} = \frac{2}{9}$$

34. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively.

The quadrilateral PQRS must be a -

(A) Parallelogram, which is neither a rhombus nor a rectangle

(B) Square

(C) Rectangle, but not a square

(D) Rhombus, but not a square

Ans.[A]

Sol. $\overrightarrow{PQ} = 6i + j$

$\overrightarrow{RS} = 6i + j$

$\overrightarrow{RQ} = i - 3j$

$\overrightarrow{SP} = i - 3j$

$\left|\overrightarrow{PQ}\right| \neq \left|\overrightarrow{RQ}\right|$ ($\therefore$ not a rhombus or a rectangle)

PQ || RS

RQ || SP

Also $\overrightarrow{PQ} . \overrightarrow{RQ} \neq 0$

$\therefore$ PQRS is not a square

$\Rightarrow$ PQRS is a parallelogram

35. The value of $\lim\limits_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\,\ell n(1+t)}{t^4+4}dt$ is

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

Ans.[B]

Sol. Use L'hospital rule in

$$\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\,\ell n(1+t)}{t^4+4}dt$$

$$= \lim_{x\to 0} \frac{x\,\ell n(1+x)}{(x^4+4)3x^2}$$

$$= \lim_{x\to 0} \frac{\ell n(1+x)}{x.3(x^2+4)}$$

$$= \frac{1}{3.4} = \frac{1}{12}$$

36. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solution, is -

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

Ans.[A]

SECTION – II
Multiple Correct Answers Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

37. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(A) $-(2+\sqrt{3})$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Ans.[B]

Sol. As sum of two sides is always greater than third side, So x > 1

Now

$$\text{Cos}\ \frac{\pi}{6} = \frac{a^2+b^2-c^2}{2ab}$$

$$\frac{\sqrt{3}}{2} = \frac{(a-b)^2+2ab-c^2}{2ab}$$

$\sqrt{3}-2 = \frac{(a-b)^2-c^2}{ab}$

$\sqrt{3}-2 = \frac{(x+2)^2-(2x+1)^2}{(x^2+x+1)\ (x^2-1)}$

$\sqrt{3}-2 = \frac{-3}{x^2+x+1}$

$x^2 + x + 1 = \frac{3}{2-\sqrt{3}}$

$x^2 + x + 1 = 3(2 + \sqrt{3})$

$x^2 + x - 5 - 3\sqrt{3} = 0$

$(x - (1 + \sqrt{3}) (x + (2 + \sqrt{3})) = 0$

$x = 1 + \sqrt{3}, \ -(2 + \sqrt{3})$

So $x = \sqrt{3} + 1$

38. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be -

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Ans.[C, D]

Sol. Let $A\left(t_1^2, 2t_1\right)$ $B\left(t_2^2, 2t_2\right)$

Slope $= \frac{2(t_2-t_1)}{t_2^2-t_1^2} = \frac{2}{t_1+t_2}$

Equation of circle will be

$(x-t_1^2)\ (x-t_2^2) + (y-2t_1)\ (y-2t_2) = 0$

$x^2+y^2 - x(t_1^2+t_2^2) - 2y(t_1+t_2) + t_1^2 t_2^2 + 4t_1t_2 = 0$

As it touches x axis so

$t_1^2 t_2^2 + 4t_1t_2 = \frac{(t_1^2+t_2^2)^2}{4}$

$4t_1^2t_2^2 + 16t_1t_2 = t_1^4 + t_2^4 + 2t_1^2t_2^2$

$\left(t_1^2 - t_2^2\right)^2 = 16\, t_1t_2$ ...(1)

AB is diameter so

$(t_1^2 - t_2^2)^2 + 4(t_1 - t_2)^2 = 4r^2$ ...(2)

From (1) and (2)

$4t_1t_2 + (t_1 - t_2)^2 = r^2$

$(t_1 + t_2)^2 = r^2$

$t_1 + t_2 = \pm r$

$\therefore$ Slope $= \pm \frac{2}{r}$

39. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are) -

(A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

Ans.[A]

Sol. $I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}$

$= \int_0^1 \frac{x^8 - 4x^7 + 6x^6 - 4x^5 + x^4}{1+x^2}dx$

$= \int_0^1 (x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2})dx$

$= \frac{22}{7} - \pi$

40. Let z_1 and z_2 be two distinct complex numbers let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg (w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(B) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

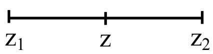
(D) Arg $(z - z_1)$ = Arg $(z_2 - z_1)$

Ans.[A,C,D]

Sol. $t = \dfrac{z - z_1}{z_2 - z_1}$

So, $\dfrac{z - z_1}{z_2 - z_1} = t\,e^{io}\ \forall\ t \in (0, 1)$

So Geometrically



So option A, C, D are true.

41. Let f be a real valued function defined on the interval $(0, \infty)$ by $f(x) = \ell n\, x + \int_0^x \sqrt{1 + \sin t}\, dt$. Then which of the following statement(s) is (are) true ?

(A) $f''(x)$ exists for all $x \in (0, \infty)$

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

(D) there exists $\beta > 0$ such that $|f'(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

Sol.[B,C]

$f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}\, dt$

$f'(x) = \dfrac{1}{x} + \sqrt{1 + \sin x} = \dfrac{1}{x} + \left|\cos\dfrac{x}{2} + \sin\dfrac{x}{2}\right|$

(A is not correct)

$\dfrac{1}{x} + \sqrt{1 + \sin x} < \ln x + \int_0^x \sqrt{1 + \sin t}\ dt$

$\because\ \ln x > \dfrac{1}{x}$ for some $x = \alpha \quad \forall\ \alpha > 1$

and $\sqrt{1+\sin x} < \int_0^x \sqrt{1+\sin t}\, dt$

for some $x = \alpha \ \forall \alpha > 1$

so option C is correct

SECTION – III
(Paragraph Type)

This section contains 2 paragraphs. Based upon the first paragraph **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question 42 to43

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B

42. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is -

(A) $2x - \sqrt{5}\ y - 20 = 0$

(B) $2x - \sqrt{5}\ y + 4 = 0$

(C) $3x - 4y + 8 = 0$

(D) $4x - 3y + 4 = 0$

Ans.[B]

Sol. $y = m(x-4) \pm 4\sqrt{1+m^2}$

$y = mx \pm \sqrt{9m^2 - 4}$

$-4m \pm 4\sqrt{1+m^2} = \pm \sqrt{9m^2-4}$

$16m^2 + 16 + 16m^2 \mp 32\, m\sqrt{1+m^2} = 9m^2 - 4$

$\mp 32m\sqrt{1+m^2} = -23m^2 - 20$

$1024m^2 + 1024\, m^4 = 529m^4 + 400 + 920\, m^2$

$495\, m^4 + 104\, m^2 - 400 = 0$

$(5m^2 - 4)\,(99m^2 + 100) = 0$

$\therefore\ m^2 = \frac{4}{5} \qquad \therefore\ m = \pm\frac{2}{\sqrt{5}}$

So tangent with positive slope

$y = \frac{2}{\sqrt{5}}x \pm \frac{4}{\sqrt{5}}$

$2x - \sqrt{5}\,y \pm 4 = 0$

43. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

Ans.[A]

Sol. $x^2 + y^2 - 8x = 0$

$4x^2 - 9y^2 = 36$

$x^2 + \left(\frac{4x^2 - 36}{9}\right) - 8x = 0$

$13x^2 - 72x - 36 = 0$

$(x - 6)(13x + 6) = 0$

$x = 6, \frac{-6}{13}$

$x = 6, y = \pm\sqrt{12}$

$\therefore$ Equation of required circle

$(x - 6)^2 + (y - \sqrt{12})(y + \sqrt{12}) = 0$

$x^2 + y^2 - 12x + 24 = 0$

Paragraph for Question 44 to 46

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots, p-1\}\right\}$$

44. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is -

(A) $(p - 1)^2$ (B) $2(p - 1)$ (C) $(p - 1)^2 + 1$ (D) $2p - 1$

Ans.[D]

Sol. $|A| = a^2 - bc$

if A is symmetric b = c

then $|A| = (a + b)(a - b)$

So a & b can attain $2(p - 1)$ solution

It A is skew symmetric then a = b = c = 0

So total no. of solution = $2p - 2 + 1 = 2p - 1$

45. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[Note : The trace of a matrix is the sum of its diagonal entries.**]**

(A) $(p - 1)(p^2 - p + 1)$ (B) $p^3 - (p - 1)^2$ (C) $(p - 1)^2$ (D) $(p - 1)(p^2 - 2)$

Ans.[C]

46. The number of A in T_p such that det (A) is not divisible by p is -

(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

Ans. **[D]**

SECTION – IV
(Integer type)

This section contains **TEN paragraphs**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

47. Let f be a real-valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to –

Ans.[9]

Sol. $Y - y = \frac{dy}{dx}(X - x)$

$\mathbf{y} - x\frac{dy}{dx} = x^3$

$x\frac{dy}{dx} - y = -x^3$

$\frac{dy}{dx} - \frac{y}{x} = -x^2$

I.F. $= \frac{1}{x}$

$\frac{y}{x} = \int -x\,dx + C$

$\frac{y}{x} = \frac{-x^2}{2} + C$

$1 = -\frac{1}{2} + C \qquad \therefore C = \frac{3}{2}$

$\therefore y = \frac{x(-x^2+3)}{2}$

$\therefore f(x) = \frac{x(-x^2+3)}{2}$

$f(-3) = \frac{-3(-9+3)}{2} = 9$

48. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0 , ± 1, ± 2 and tan θ = cot 5 θ as well as sin 2 θ = cos 4 θ is –

Ans.[3]

Sol. $\tan\theta = \cot 5\theta \Rightarrow \theta = (2n+1)\frac{\pi}{12}$

So $\theta = \pm\frac{\pi}{12}, \pm\frac{\pi}{4}, \pm\frac{5\pi}{12}$

$\therefore$ Sin 2θ = cos 4θ $\Rightarrow$ sin 2θ = $\frac{1}{2}$ or – 1

only $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{-\pi}{4}$ satisfies the given conditions

So total number of solution = 3

49. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Sol.[2] $f(\theta) = \dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$

$= \dfrac{1}{1 + 4\cos^2\theta + 3\sin\theta\cos\theta}$

$= \dfrac{1}{1 + 2(1+\cos 2\theta) + \frac{3}{2}\sin 2\theta}$

$= \dfrac{1}{3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta}$

So $f(\theta)_{max} = \dfrac{1}{3 - \sqrt{4 + \frac{9}{4}}} = 2$

50. If $\vec{a}$ and $\vec{b}$ are vector is space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\hat{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is

Ans.[5]

Sol. $|\bar{a}| = |\bar{b}| = 1 \,\&\, \bar{a}\cdot\bar{b} = 0$

$(2\,\bar{a} + \bar{b})\cdot[(\bar{a}\times\bar{b})\times(\bar{a} - 2\bar{b})]$

$= (2\,\bar{a} + \bar{b})\cdot[\bar{b} + 2\bar{a}] = |\,\bar{b}\,|^2 + 4\,|\,\bar{a}\,|^2 = 5$

51. The line 2x + y = 1 is tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Ans.[2]

Sol. $1 = 4a^2 - b^2$ ... (1)

$\dfrac{2a}{e} = 1$

$a = \dfrac{e}{2}$ ... (2)

also $b^2 = a^2(e^2 - 1)$ ... (3)

(1) & (3)

$1 = 4a^2 - a^2e^2 + a^2 \Rightarrow 1 = 5a^2 - a^2e^2$

$\Rightarrow 1 = \frac{5e^2}{4} - \frac{e^4}{4}$

$\Rightarrow e^4 - 5e^2 + 4 = 0$

$\Rightarrow (e^2 - 4)(e^2 - 1) = 0$

$\therefore$ e = 2

52. If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then | d | is –

Ans.[6]

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

$= \hat{i}(-1) - \hat{j}(-2) + \hat{k}(-1)$

Plane is normal to vector $\hat{i} - 2\hat{j} + \hat{k}$

$1(X - 1) - 2(Y - 2) + 1(Z - 3) = 0$

$X - 2Y + Z = 0$

$\sqrt{6} = \frac{|d|}{\sqrt{6}} \Rightarrow |d| = 6$

53. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

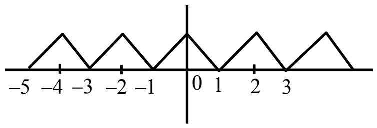
$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x \, dx$ is

Ans.[4]

Sol. $f(x) = \begin{cases} \{x\} & \forall [x] \text{ is odd} \\ 1 = \{x\} & \forall [x] \text{ is even} \end{cases}$

graph of y = f(x) is



$\because$ f(x) & cos πx both are even functions

So, $I = \frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x \, dx$

$= \frac{\pi^2}{5} \int_{0}^{10} f(x) \cos(\pi x) dx$

$\therefore$ f(x) & cos πx both are periodic then

$I = \pi^2 \int_0^2 f(x) \cos(\pi x) \, dx$

$= \pi^2 \left[\int_0^1 (1-x) \cos x(\pi x) dx + \int_1^2 (x-1) \cos(\pi x) dx \right]$

$= \pi^2 \left[\frac{2+2}{\pi^2} \right] = 4$

54. Let ω be the complex number $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$. Then the number of distinct complex number z satisfying

$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to =

Ans.[1]

Sol. On solving the determinant

It become $z^3 = 0$

So no. of solutions = 1

55. Let S_k, k = 1, 2, ……, 100, denote the sum of the infinite geometric series whose first term is $k\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is –

Ans.[4]

Sol. $S_k = \frac{K}{\lfloor K}$

$\sum_{K=1}^{100} |(k^2 - 3k + 1)S_k|$

$= 1 + 1 + \sum_{K=3}^{100}\left|\frac{(k^2-3k+1)}{\lfloor k-1}\right|$

$= 2 + \sum\left|\frac{k-1}{\lfloor k-2} - \frac{k}{\lfloor k-1}\right|$

$= 2 + 2 - \frac{100}{\lfloor 99} = 4$

56. The number of all possible values of θ, where 0 < θ < π, for which the system of equations

(y + z) cos 3 θ = (xyz) sin 3θ

$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

(xyz) sin3θ = (y + 2z) cos 3θ + y sin 3θ)

have a solution (x_0, y_0, z_0) with $y_0z_0 \neq 0$, is

Ans.[3]

Sol. (xyz) sin 3θ + y (– cos 3θ) + z (– cos 3θ) = 0

(xyz) sin 3θ + y (–2 sin 3θ) + z (–2 cos 3θ) = 0

(xyz) sin 3θ + y (– cos 3θ – sin 3θ) + z (– 2cos 3θ) = 0

For $y_0z_0 \neq 0 \Rightarrow$ Nontrivial solution

$$\begin{vmatrix} \sin 3\theta & -\cos 3\theta & -\cos 3\theta \\ \sin 3\theta & -2\sin 3\theta & -2\cos 3\theta \\ \sin 3\theta & -\cos 3\theta - \sin 3\theta & -2\cos 3\theta \end{vmatrix} = 0$$

$$\sin 3\theta \cos 3\theta \begin{vmatrix} 1 & \cos 3\theta & 1 \\ 1 & 2\sin 3\theta & 2 \\ 1 & \cos 3\theta + \sin 3\theta & 2 \end{vmatrix} = 0$$

$\sin 3\theta \cos 3\theta$ [(4sin 3θ – 2 cos 3θ – 2sin 3θ) – (2cos 3θ – cos 3θ – sin 3θ) + 2 cos 3θ – 2 sin 3θ] = 0

$\Rightarrow$ (sin3θ cos3θ) [2 sin3θ – 2cos 3θ – cos 3θ + sin 3θ + 2cos 3θ – 2sin 3θ] = 0

$\Rightarrow$ (sin3θ cos3θ) (sin3θ – cos 3θ) = 0

$\Rightarrow \sin 3\theta = 0 \quad \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow \cos 3\theta = 0 \quad \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$

These two donot satisfy system of equations

$\Rightarrow \sin 3\theta = \cos 3\theta \ \Rightarrow 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4} = 3$

No. of solutions = 3

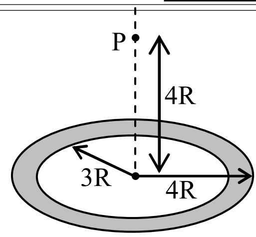
Part –III (PHYSICS)

SECTION – I

Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

57. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is-



(A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

Ans. **[A]**

Sol. $\Delta W_{ext} = U_2 - U_1$

for unit +ve mass

$U_1 = V_1$ and $U_2 = V_2 = 0$

$$V_1 = \int dV = \int -\frac{Gdm}{(r^2+16R^2)^{1/2}}$$

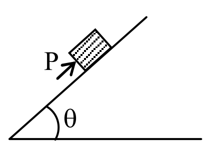
$$= \int -\frac{GM2\pi rdr}{7\pi R^2(r^2+16R^2)^{1/2}}$$

Put $r^2 + 16R^2 = t^2$

$$V_1 = -\frac{2GM}{7R^2}\int_{5R}^{4\sqrt{2}R} dt = -\frac{2GM}{7R}(4\sqrt{2}-5)$$

$$\Delta W_{ext} = U_2 - U_1 = \frac{2GM}{7R}(4\sqrt{2}-5)$$

58. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tan θ > μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like –



(A

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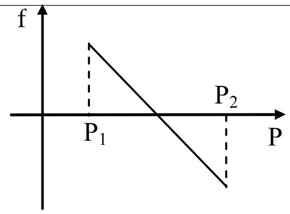

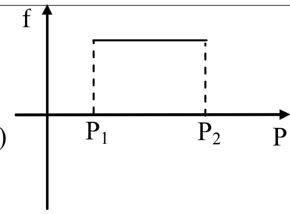
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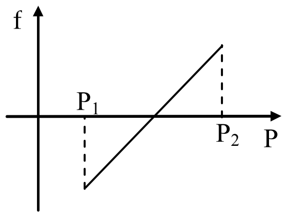
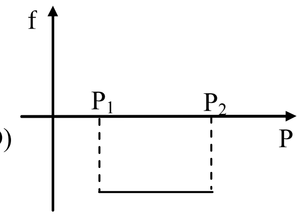

(C)


(D)

Ans. **[A]**

Sol. In the given range block is in equilibrium so

$P - mg \sin\theta + f = 0$

$f = mg \sin\theta - P$

Equation of straight line with negative slope.

59. A real gas behaves like an ideal gas if its -

(A) pressure and temperature are both high

(B) pressure and temperature are both low

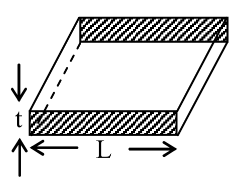
(C) pressure is high and temperature is low

(D) pressure is low and temperature is high

Ans. **[D]**

Sol. **Reason :** PV = nRT holds true in case of low pressure and high temperature conditions.

60. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is-



(A) directly proportional to L

(B) directly proportional to t

(C) independent of L

(D) independent of t

Ans. **[C]**

Sol. $R = \frac{\rho L}{A} = \frac{\rho L}{Lt}$

$R = \frac{\rho}{t}$

R is independent of L

61. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is-

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (B) $R_{100} = R_{40} + R_{60}$ (C) $R_{100} > R_{60} > R_{40}$ (D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} + \frac{1}{R_{40}}$

Ans. **[D]**

Sol. Rated power = $\frac{V^2}{R}$

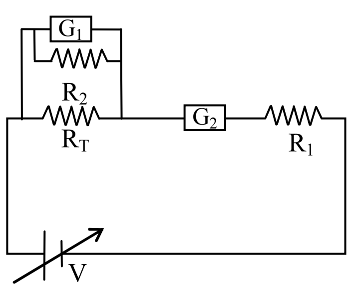
$R \propto \frac{1}{\text{Rated power}}$

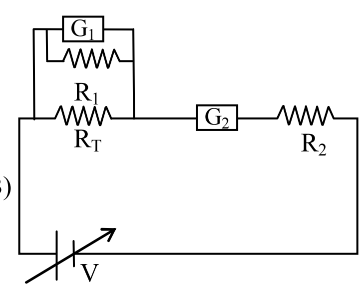
$P_1 > P_2 > P_3$

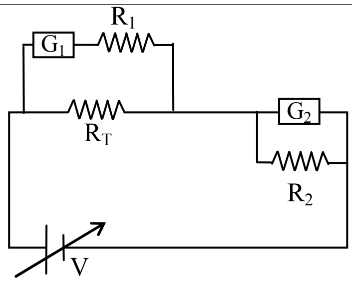
$\therefore \frac{1}{R_1} > \frac{1}{R_2} > \frac{1}{R_3}$

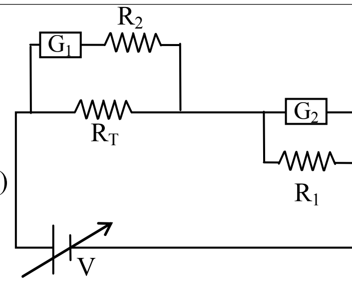
$\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

62. To verify Ohm's law, student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct to carry out the experiment is-

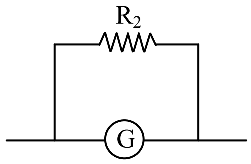
(A)


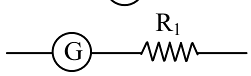
(B)


(C) 

(D) 

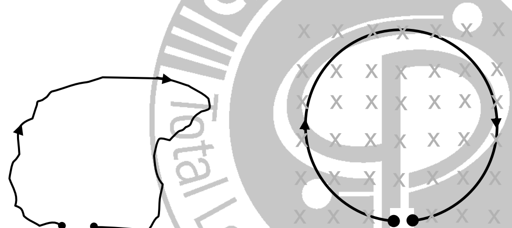
Ans. **[C]**

Sol. Converted ammeter = 

Converted ammeter = 

Voltmeter should be connected in parallel to R_T and Ammeter should be connected in series with R_T.

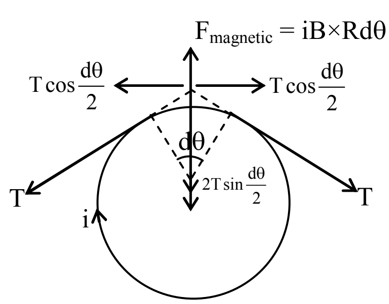
63. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current i in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into plane of the paper, the wire takes the shape of a circle. The tension in the wire is-



(A) IBL (B) $\frac{IBL}{\pi}$ (C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

Ans. **[C]**

Sol.



T = tension

$iB \times R\, d\theta = 2T \sin\frac{d\theta}{2}$

$T = iBR$

$2\pi R = L$

$R = \frac{L}{2\pi}$ $\quad \therefore T = \frac{iBL}{2\pi}$

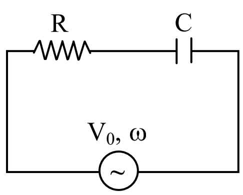
64. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased -

(A) the bulb glows dimmer (B) the bulb glows brighter

(C) total impedance of the circuit is unchanged (D) total impedance of the circuit increases

Ans. **[B]**

Sol.



$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

ω increased z decreased

$\therefore$ current in circuit increase

$\therefore$ Bulb glow brighter.

SECTION – II
Multiple Correct Answers Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

65. A student uses a simple pendulum 1 m length to determine g, the acceleration due to gravity. He uses a stop watch the least count of 1 sec for this records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?

(A) Error ΔT in measuring T, the time period, is 0.05 seconds

(B) Error ΔT in measuring T, the time period, is 1 second

(C) Percentage error in the determination of g is 5%

(D) Percentage error in the determination of g is 2.5 %

Ans. **[A,C]**

Sol. Time period (T) = $\frac{\text{Total time(t)}}{\text{no. of oscillations}}$

So $\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1\,\text{sec}}{40\,\text{sec}}$

$\Delta T = \frac{1}{40} \times 2 = 0.05$ sec

$T = 2\pi\sqrt{\frac{\ell}{g}}$; $T^2 = \frac{4\pi^2 \ell}{g}$; $g = \frac{4\pi^2 \ell}{T^2}$;

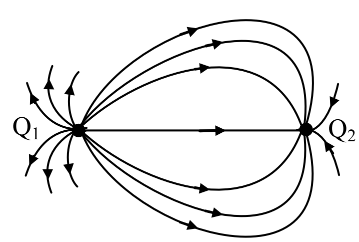
$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$; $\frac{\Delta \ell}{\ell} = 0$

$\Delta T = 0.05$ sec ; $T = 2$ sec.

Putting we get $\frac{\Delta \ell}{\ell} = \frac{2 \times 0.05}{2} = 0.05$

$\frac{\Delta g}{g} \times 100 = 5\ \%$

66. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(A) $|Q_1| > |Q_2|$

(B) $|Q_1| < |Q_2|$

(C) at a finite distance to the left of Q_1 the electric field is zero

(D) at a finite distance to the right of Q_2 the electric field is zero

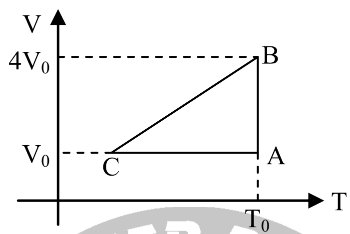
Ans. **[A,D]**

Sol. Number of field lines emitting from Q_1 is more than number of field lines reaching at Q_2

So $|Q_1| > |Q_2|$

and if so $\vec{E}$ at a point which is right to Q_2 will be zero.

67. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, As shown in figure. Its pressure at A is P_0. Choose the correct option(s) from the following



(A) Internal energies at A and B are the same

(B) Work done by the gas in process AB is $P_0V_0 \ell n\, 4$

(C) Pressure at C is $P_0/4$

(D) Temperature at C is $\frac{T_0}{4}$

Ans. **[A,B]**

Sol. From figure

AB $\rightarrow$ isothermal process

So $T_A = T_B \Rightarrow$ Internal energies will be same.

$$W_{AB} = nRT_0\, ln\left(\frac{V_2}{V_1}\right) = P_0V_0\, ln\, 4$$

It is not given that line BC passes through origin. So we can't find pressure or temperature at point C.

68. A Point mass 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ?

(A) Total momentum of the system is 3 kg ms^{-1}

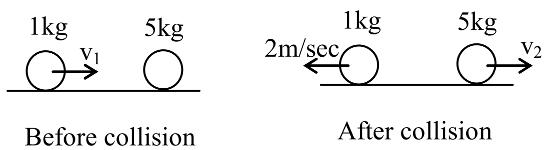
(B) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(C) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4 J

Ans. **[A,C]**

Sol.



Collision is elastic so

$v_2 + 2 = v_1$ ......(i)

Conservation of momentum,

$1 \times v_1 + 0 = -2 \times 1 + 5 \times v_2$ ....(ii)

Solving

$v_1 = 3$ m/sec

$v_2 = 1$ m/sec

Total momentum $\vec{p}_{system} = 1 \times v_1 = 3$ kg-m/sec

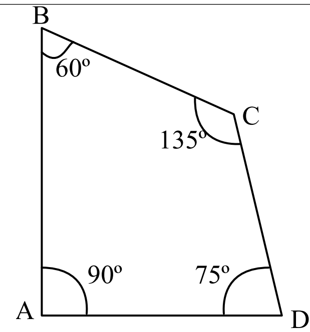
Momentum of 5 kg = $5 \times v_2 = 5$ kg-m/sec

$v_{CM} = \dfrac{1\times 3 + 5\times 0}{6} = 0.5$ m/sec

$k_{CM} = \dfrac{1}{2}\times(M_1 + M_2)v_{CM}^2 = 0.75$ joule

$k_{system} = \dfrac{1}{2} \times 1 \times 9 = 4.5$ joule.

69. A rat OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60º (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct ?



(A) The ray gets totally internally reflected at face CD

(B) The ray comes out through face AD

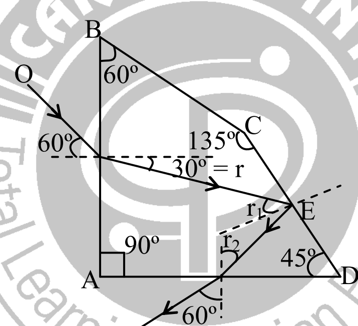
(C) The angle between the incident ray and the emergent ray is 90º

(D) The angle between the incident ray and the emergent ray is 120º

Ans. **[A,B,C]**

Sol. Refraction at first surface AB : $\frac{\sin 60^\circ}{\sin r} = \frac{\sqrt{3}}{1}$

r = 30º



it hits at E

By geometry angle $r_1 = 45^\circ$, $r_2 = 45^\circ$

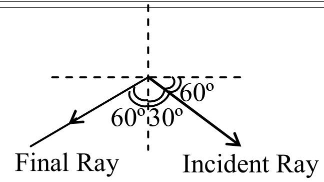
We know $\sin\theta_C = \frac{1}{\sqrt{3}}$ and $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}}$$

so $45^\circ > \theta_C$

So total internal reflection occurs.

After reflection angle of incidence at AD will be 30º so ray comes out making an angle 60º with the normal at AD.

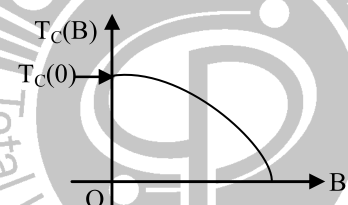


SECTION – III
Comprehension Type

This section contains 2 paragraphs.. Based upon the first paragraph 2 multiple choice question and based upon the second paragraph 3 multiple choice question have to be answered. Each of these questions has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Questions 70 to 71

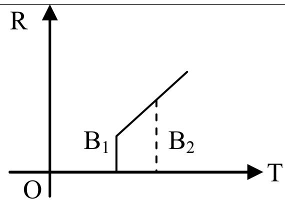
Electrical resistance of certain materials, knows as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than T_C (0) if they are placed in a magnetic field, i.e. the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of T_C (B) on B is shown in the figure.

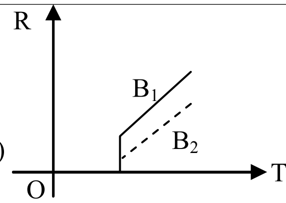


70. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields ?

(C

)



(D) 

Ans. **[A]**

Sol. $B_2 > B_1$

so $T_C(B_2) < T_C(B_1)$

Dashed line   Solid line

Resistance $\propto$ Temperature above critical .

71. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5. Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(A) B = 5 Tesla, T_C (B) = 80 K
(B) B = 5 Tesla, 75 K < T_C (B) < 100 K
(C) B = 10 Tesla, 75 K < T_C (B) < 100 K
(D) B = Tesla, T_C(B) = 70 K

Ans. **[B]**

Sol. $T_C(0) =$ 100 K, B = 0

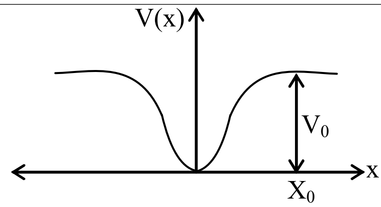
B = 7.5 tesla

T_C (B) = 75

If B = 5 Tesla, T_C(B) should be greater than 75 K.

Paragraph for Questions 72 to 74

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for | x | near the origin and becomes a constant equal to V_0 for $| x | \geq X_0$ (see figure)



72. If the total energy of the particle is E, it will perform periodic motion only if-

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

Ans. **[C]**

Sol. Energy Total should be less than maximum potential energy

so $E < V_0$ and $E > 0$.

73. For periodic motion of small amplitude A, the time period T of this particle is proportional to-

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Ans **[B]**

Sol. $V(x) = \alpha x^4$

$$[\alpha] = \frac{[V(x)]}{[x]^4} = \frac{[ML^2T^{-2}]}{[L^4]} = [ML^{-2}T^{-2}]$$

Time period $\propto$ (Amplitude)^x $(\alpha)^y$ (Mass)^z

$[T] = [L]^x [ML^{-2}T^{-2}]^y [M]^2$

Solving $x = -1, \quad y = -\frac{1}{2}, \quad z = \frac{1}{2}$

$$T = A^{-1}\ \alpha^{-1/2}\ M^{1/2} = \frac{1}{A}\sqrt{\frac{M}{\alpha}}$$

74. The acceleration of this particle for $|x| > X_0$ is -

(A) proportional to V_0

(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) zero

Ans. **[D]**

Sol. for $|x| > x_0$

$U = \text{constant}$

$$F = -\frac{dU}{dx} = 0$$

acceleration = zero.

SECTION – IV

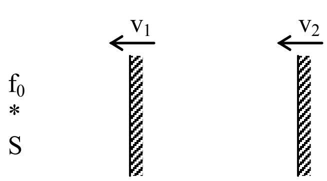
Integer Type

This section contains **TEN questions.** The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

75. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the car in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 m/s.

Ans. [7]

Sol.



$$f_1 = f_0\left(\frac{v+v_1}{v}\right)$$

$$f_2 = f_1\left(\frac{v}{v-v_1}\right) = \frac{f_0(v+v_1)}{(v-v_1)}$$

$$f_2' = \frac{f_0(v+v_2)}{(v-v_2)}$$

$$\frac{f'_2 - f_2}{f_0} = \frac{(v+v_2)}{(v-v_2)} - \frac{(v+v_1)}{(v-v_1)} = 0.012$$

$$\frac{(v+v_2)(v-v_1)-(v+v_1)(v-v_2)}{(v-v_2)(v-v_1)} = 0.012$$

$$\frac{v^2+v(v_2-v_1)-v_1v_2-v^2+v_1v_2-v(v_1-v_2)}{(v-v_2)(v-v_1)} = 0.012$$

$$\frac{2v(v_2-v_1)}{(v-v_1)(v-v_2)} = 0.012$$

$$\frac{2}{v}(v_2-v_1) = 0.012$$

$$(v_2 - v_1) = 0.006 \times 330 \times \frac{18}{5}$$

$$= 7.128$$

76. The focal length of thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is ?

Ans. [**6**]

Sol. $m = \frac{f}{f+u}$

$m_{25} = \frac{20}{20-25} = -4$; $m_{50} = \frac{20}{20-50} = -\frac{2}{3}$

$\frac{m_{25}}{m_{50}} = 6$

77. An α-particle and a proton are accelerated from rest by a potential difference of 100V. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer is ?

Ans. **[3]**

Sol. After accelerating through V_0 KE of a particle becomes = qV_0 evolts

so

$KE_\alpha = 200$ eV

$KE_p = 100$ eV

$\lambda_{debroglied} = \frac{h}{\sqrt{2MKE}}$

$\frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{M_\alpha KE_\alpha}{M_p KE_p}} = \sqrt{\frac{4\times 200}{1\times 100}} = 2\sqrt{2}$

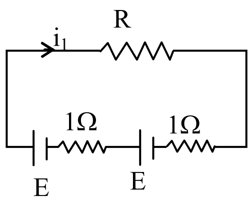
$= 2 \times 1.414$

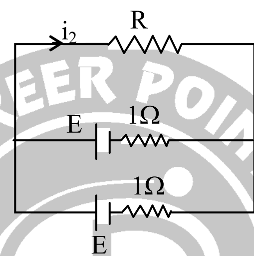
$\cong 3$

78. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R is Ω is ?

Ans. **[4]**

Sol.





$i_1 = \frac{2E}{R+2}$

$J_1 = \left(\frac{2E}{R+2}\right)^2 .R$

$E_{eq} = E$

$r_{eq} = 0.5\Omega$

$i_2 = \frac{E}{R+0.5}$

$J_2 = \left(\frac{E}{R+0.5}\right)^2 R$

From $J_1 = 2.25\ J_2$

$\left(\frac{2E}{R+2}\right)^2 R = 2.25 \left(\frac{E}{R+0.5}\right)^2 R$

$\frac{2}{R+2} = \frac{1.5}{R+0.5}$

$2R + 1 = 1.5\,R + 3$

$0.5\,R = 2$

$R = \frac{2}{0.5} = 4\,\Omega$

79. Two spherical bodies A(radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ?

Ans. **[9]**

Sol. $\lambda_A = 500$ nm

$\lambda_B = 1500$ nm

$\lambda_A T_A = \lambda_B T_B$

$\frac{T_1}{T_2} = \frac{T_A}{T_B} = 3$ … (i)

$\frac{r_A}{r_B} = \frac{1}{3}$

From Stefan's law

$$\frac{E_A}{E_B} = \frac{\sigma(4\pi r_{A^2})(T_1^4)}{\sigma(4g r_{B^2})(T_2^4)} = \left(\frac{1}{3}\right)^2 \times (3)^4 = 9$$

80. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is ?

Ans. **[5]**

Sol. $y_1 = 4\sin(2x - 6t)$

$y_2 = 3\sin(2x - 6t - \pi/2)$

$\phi = \frac{\pi}{2}$

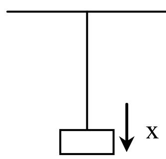
$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A_{res} = \sqrt{3^2 + 4^2 + 0} = 5$

81. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad/s. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is ?

Ans. **[4]**

Sol.



$\text{strain} = \frac{x}{\ell}$

$$\frac{\text{stress}}{\text{strain}} = Y$$

stress = Yx $\quad (\ell = 1 \text{ m})$

$$\frac{F}{A} = Yx$$

F = Ayx

Ayx = ma

$$a = \frac{AYx}{m} \quad ; \quad \omega = \sqrt{\frac{AY}{m}}$$

$$140 = \sqrt{\frac{4.9\times10^{-7}\times n\times10^{9}}{0.1}}$$

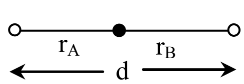
$$140 = 70\sqrt{n}$$

n = 4

82. A binary star consists of two stars (mass 2.2 M_S) and B(mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is ?

Ans. **[6]**

Sol.



$L_A = m_A\omega\, r_A^2$

$L_B = m_B\omega r_B^{\,2}$

Ratio (K) = $\dfrac{L_A + L_B}{L_B} = \dfrac{L_A}{L_B} + 1 = \dfrac{m_A r_A^{\,2}}{m_B r_B^{\,2}} + 1$

$\dfrac{r_A}{r_B} = \dfrac{m_B}{m_A} = \dfrac{11}{2.2} = 5$

Ratio (K) = $\dfrac{1}{5} \times 5^2 + 1 = 6$

83. Gravitational acceleration on the surface of a planet is $\dfrac{\sqrt{6}}{11}$g, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\dfrac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km/s, the escape speed on the surface of the planet in km/s will be ?

Ans. **[3]**

Sol. $\dfrac{\upsilon_p}{\upsilon_c} = \sqrt{\dfrac{2g_pR_p}{2g_eR_e}} = \sqrt{\dfrac{g_p}{g_e} \times \dfrac{R_p}{R_e}}$ … (1)

$\Rightarrow \dfrac{M_p}{\frac{4}{3}\pi R^3{}_p} = \dfrac{2}{3}\dfrac{M_e}{\frac{4}{3}\pi R_e^3} \Rightarrow \dfrac{M_P}{M_e} = \dfrac{2}{3}\dfrac{R_p^3}{R_e^3}$ … (2)

$\dfrac{GM_p}{R_p^2} = \dfrac{\sqrt{6}}{11}\dfrac{GM_e}{R_e^{\,2}} \Rightarrow \dfrac{M_p}{M_c} = \dfrac{\sqrt{6}}{11}\dfrac{R_p^2}{R_e^2}$ … (3)

from (2) and (3)

$\dfrac{2}{3}\dfrac{R_p^3}{R_e^3} = \dfrac{\sqrt{6}}{11}\dfrac{R_p^2}{R_e^2} \Rightarrow \dfrac{R_p}{R_e} = \dfrac{3\sqrt{6}}{22}$ … (4)

from (1) and (4)

$\dfrac{\upsilon_p}{\upsilon_e} = \sqrt{\dfrac{\sqrt{6}}{11} \times \dfrac{3\sqrt{6}}{22}} = \sqrt{\dfrac{18}{242}} = \dfrac{3}{11}$

v_p = 3 km /sec.

84. A piece of ice (heat capacity = 2100 $Jkg^{-1}{}^{o}C^{-1}$ and latent heat = 3.36×10^8 J kg^{-1}) of mass m grams is at –5ºC at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is ?

Ans. **[8]**

Sol. The amount of heat required to raise the temp from –5°C to 0°C.

$Q_1 = m \times 2100 \times 10^{-3} \times 5 = 10.5$ m Joule

The amount of heat required to melt 1 gm

ice = $10^{-3} \times 3.36 \times 10^5 = 336$ J

420 = 336 + 10.5 m

10.5 = 84

m = 8 gm.