Part – I (CHEMISTRY)

SECTION – I

CODE - 2 | Straight Objective Type | 11/04/2010

This section contains 6 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule B_2 is

(A) 1 and diamagnetic (B) 0 and diamagnetic

(C) 1 and paramagnetic (D) 0 and paramagnetic

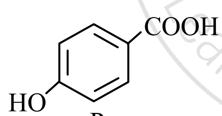
Ans. **[A]**

Sol. $\sigma_{1s^2}\ \ \sigma^*{}_{1s^2}\ \ \sigma_{2s^2}\ \ \sigma^*{}_{2s^2}\ \ \pi^2_{2p_x} = \pi_{2p_y}$

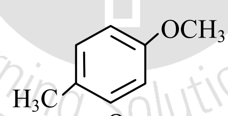
$$B.O = \frac{1}{2}(6-4) = 1$$

Diamagnetic

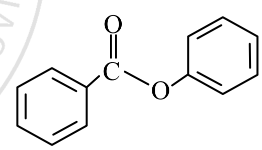
2. The compounds P, Q and S



P



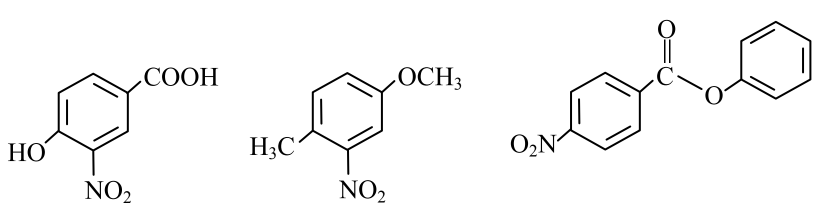
Q



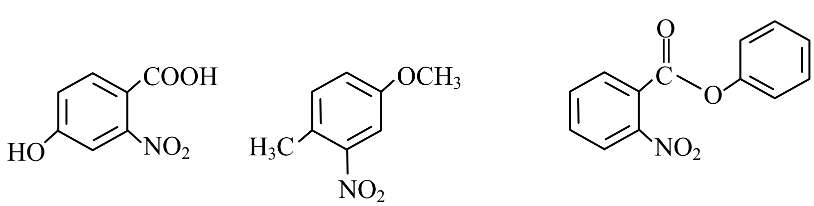
S

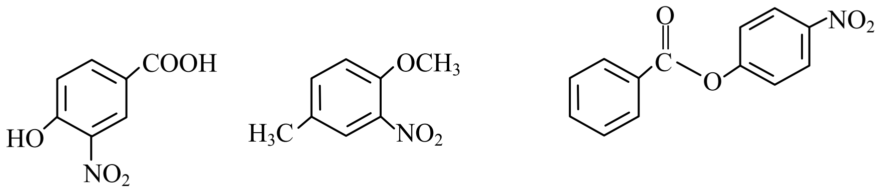
were separately subjected to nitration using HNO_3/H_2SO_4 mixture. The major product formed in each case respectively, is

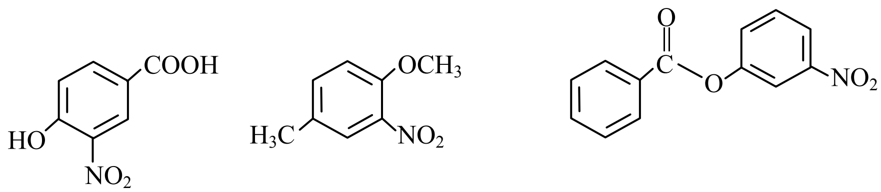
(A)



(B)

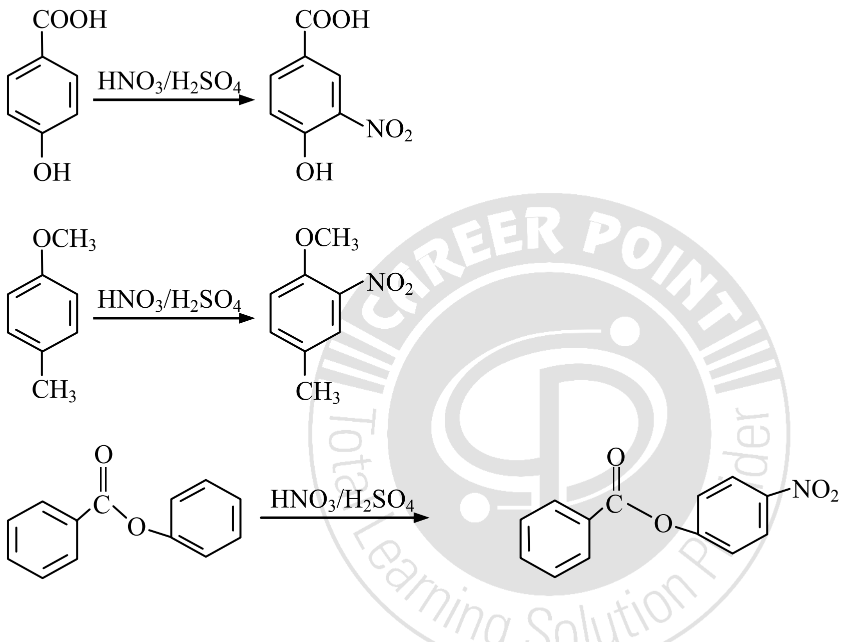


(C) 

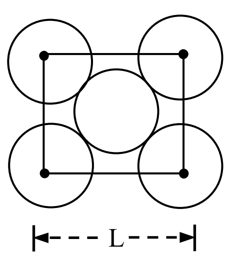
(D) 

Ans. **[C]**

Sol.



3. The packing efficiency of the two-dimensional square unit cell shown below is

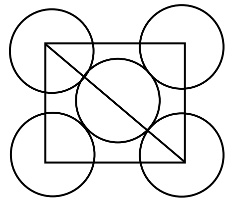


(A) 39.27 % (B) 68.02%

(C) 74.05% (D) 78.54%

Ans. **[D]**

Sol. Area of square $= L^2$



& $4R = \sqrt{2}\, L$

$$R = \frac{L}{2\sqrt{2}}$$

$$\%\text{ packing efficiency } (\eta) = \frac{\text{area of circle}}{\text{area of square}} \times 100$$

$$= \frac{2 \times \pi\, R^2}{L^2} \times 100$$

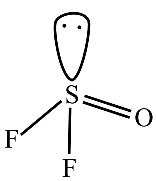
$$= \frac{2 \times \pi \dfrac{L^2}{4 \times 2}}{L^2} \times 100 = 78.5\%$$

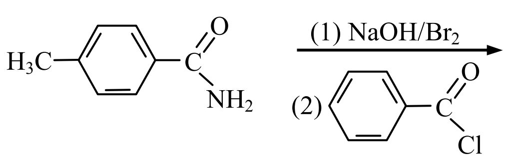
4. The species having pyramidal shape is

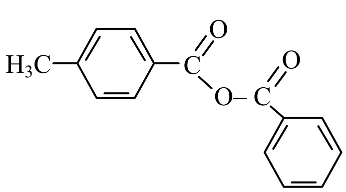
(A) SO_3 (B) BrF_3 (C) SiO_3^{2-} (D) OSF_2

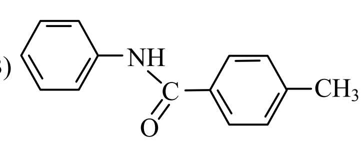
Ans. **[D]**

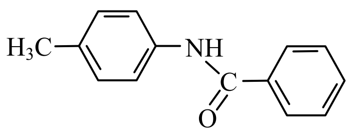
Sol.

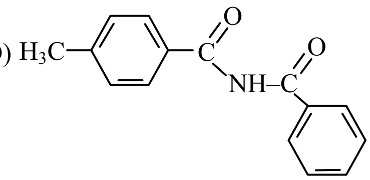


5. In the reaction  **T** the structure of the product T is

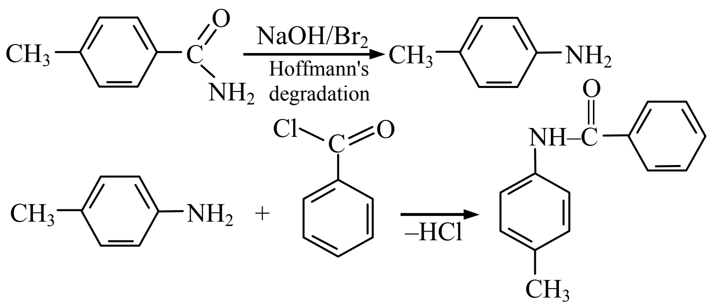
(A) 

(B) 

(C) 

(D) 

Ans. **[C]**

Sol. 

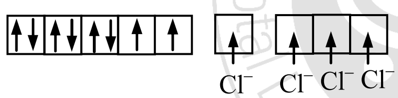
6. The complex showing a spin-only magnetic moment of 2.82 B.M. is

(A) $Ni(CO)_4$ (B) $[NiCl_4]^{2-}$ (C) $Ni((PPh_3)_4$ (D) $[Ni(CN)_4]^{2-}$

Ans. **[B]**

Sol. $[NiCl_4]^{-2}$

Ni^{+2} $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3d^8\ \ 4s^{\circ}$



hyb = sp^3

no. of unpaired electrons = 2

$\mu = \sqrt{2(4)} = \sqrt{8} = 2.82$ BM

SECTION – II

Integer Type

This section contains a group of **5** questions. The answer to each questions is a **single digit integer** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

7. Silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 g cm^{-3}. The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as $y \times 10^x$. The value of x is

Ans. **[7]**

Sol. $d = \frac{m}{V} \Rightarrow 10.5$ g/cc

Number of atoms of Ag in 1cc $\Rightarrow \frac{10.5}{108} \times N_A$

In 1cm, number of atoms of Ag = $\sqrt[3]{\frac{10.5}{108} \times N_A}$

In $1cm^2$, number of atoms of Ag = $\left(\frac{10.5}{108} \times N_A\right)^{2/3}$

In 10^{-12} m^2 or 10^{-8} cm^2, number of atoms of

$$Ag = \left(\frac{10.5}{108} N_A\right)^{2/3} \times 10^{-8} = \left(\frac{10.5 \times 6.02 \times 10^{23}}{108}\right)^{2/3} \times 10^{-8}$$

$$= 1.5 \times 10^7$$

Thus, x = 7

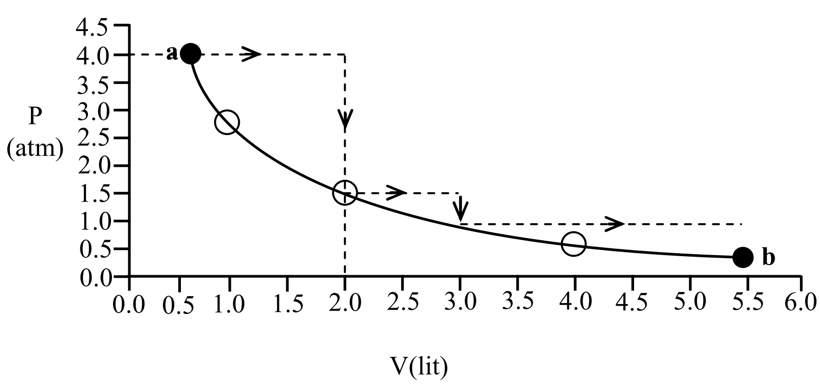
8. Among the following, the number of elements showing only one non-zero oxidation state is

O, Cl, F, N. P, Sn, Tl, Na, TI

Ans. **[2]**

Sol. F & Na only show one non zero oxidation state that are – 1 & + 1 respectively.

9. One mole of an ideal gas is taken from **a** to **b** along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is w_s and that along the dotted line paths is w_d, then the integer closest to the ratio $w_d \backslash w_s$ is



Ans. **[2]**

Sol. For solid line path show approxy isothermal process

$\therefore$ work done $|W_S| = 2.303\ (PV)\ \ \log \dfrac{5.5}{.5}$

$= 2.303 \times 4 \times .5 \times \log 11$

$\simeq 4.79$

for dashed line path work done

$w_d = 4 \times |2 - .5| + 1 \times |3 - 2| + .5 \times |5.5 - 3|$

$= 6 + 1 + 1.25 = 8.25$

$\therefore \dfrac{w_d}{w_s} = \dfrac{8.25}{4.8} = 1.71 \simeq 2$

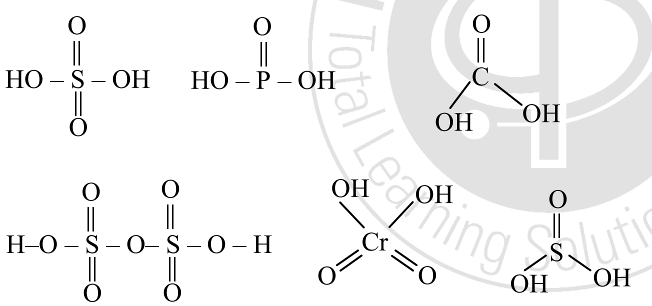
10. The total number of diprotic acids among the following is

H_3PO_4 H_2SO_4 H_3PO_3 H_2CO_3 $H_2S_2O_7$

H_3BO_3 H_3PO_2 H_2CrO_4 H_2SO_3

Ans. **[6]**

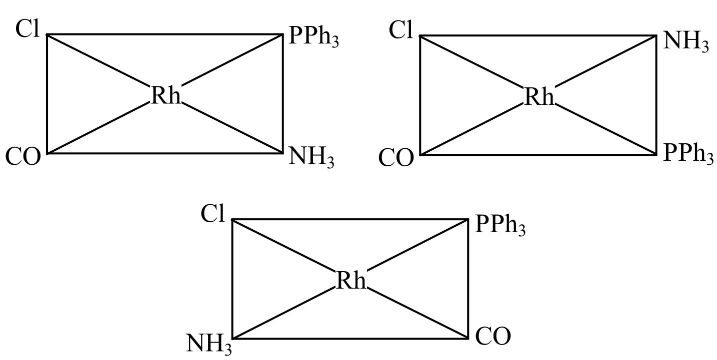
Sol.



11. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is

Ans. **[3]**

Sol.



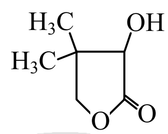
SECTION – III

Paragraph Type

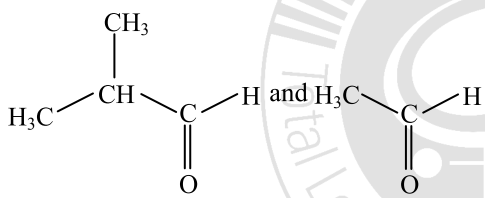
This section contains **2 paragraphs**. Based upon each of the paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

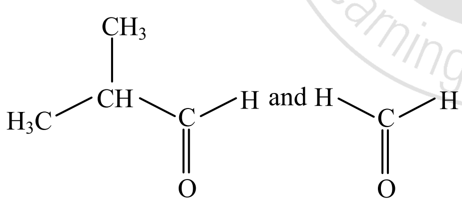
Paragraph for questions 12 to 14

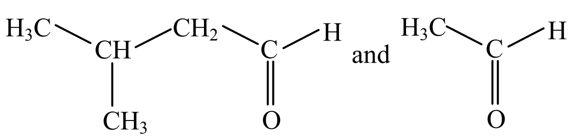
Two aliphatic aldehydes P and Q react in the presence of aqueous K_2CO_3 to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below –

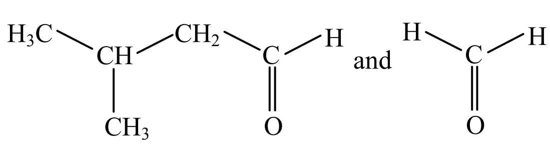


12. The compounds P and Q respectively are -

(A) 

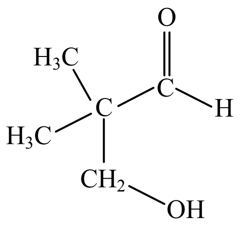
(B) 

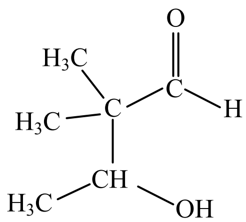
(C) 

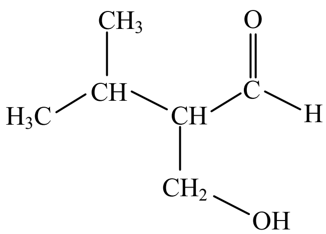
(D) 

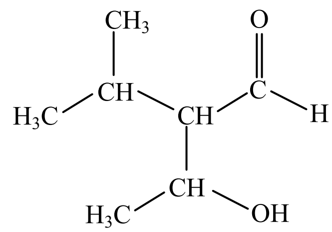
Ans. **[B]**

13. The compound R is

(A) 

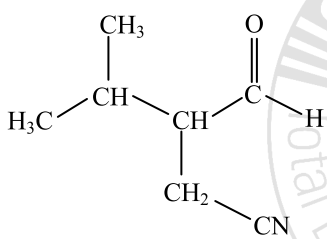
(B) 

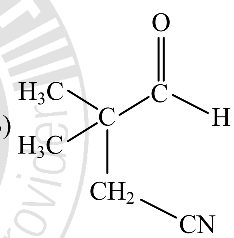
(C) 

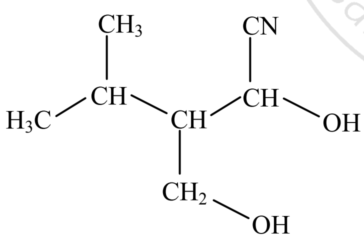
(D) 

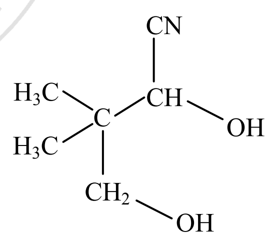
Ans. **[A]**

14. The compound S is

(A) 

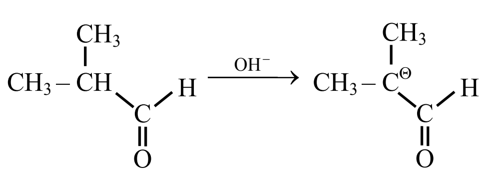
(B) 

(C) 

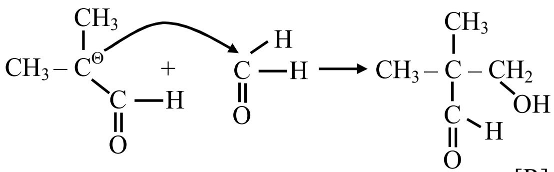
(D) 

Ans. **[D]**

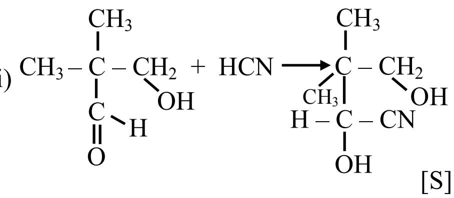
Sol. (12 to 14)

(i) 

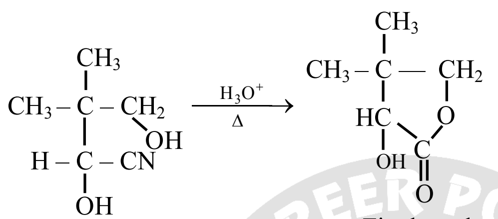
(ii)



(iii)



(iv)



Final product

Paragraph for Questions 15 to 17

The hydrogen like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

15. The state S_1 is –

(A) 1s (B) 2s (C) 2p (D) 3s

Ans. **[B]**

Sol. $\because$ One radial node

$\therefore n - \ell - 1 = 1$

or $n - \ell = 2$

$\ell = 0$

$n = 2$

Orbital name = 2s

16. Energy of the state S_1 in units of the hydrogen atom ground state energy is –

(A) 0.75 (B) 1.50 (C) 2.25 (D) 4.50

Ans. **[C]**

Sol. S_1 = Energy of e of H in ground state × $\frac{3^2}{2^2}$

= 2.25 × energy of e of H in ground state

17. The orbital angular momentum quantum number of the state S_2 is –

(A) 0 (B) 1 (C) 2 (D) 3

Sol. **[B]**

For $S_2 = n - \ell - 1$

$n - \ell = 2$

$n = 3, \ell = 1$

Orbital = 3p $\therefore \ \ell = 1$

[S_2 = energy of e of H in ground state × $\frac{3^2}{n^2}$, n = 3]

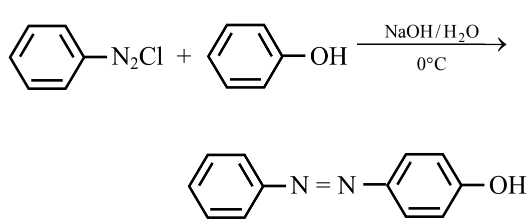
SECTION – IV

Matrix Type

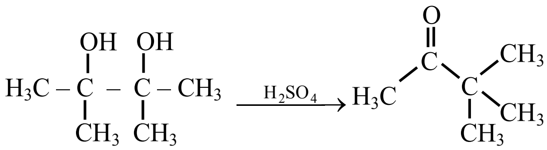
This Section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

18. Match the reactions in Column I with appropriate options in **Column II**.

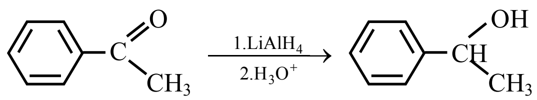
Column I **Column II**

(A) 

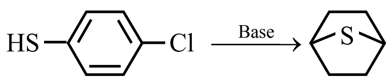
(p) Racemic mixture

(B) 

(q) Addition reaction

(C) 

(r) Substitution reaction

(D) 

(s) Coupling reaction

(t) Carbocation intermediate

Ans. [A → r,s,t; B → t; C → p,q; D → r]

19. All the compounds listed in Column I react with water. Match the result of the respective reactions with the appropriate options listed in Column II.

Column I	**Column II**
(A) $(CH_3)_2SiCl_2$	(p) Hydrogen halide formation
(B) XeF_4	(q) Redox reaction
(C) Cl_2	(r) Reacts with glass
(D) VCl_5	(s) Polymerization
	(t) O_2 formation

Ans. [A → p,s; B → p,q,r,t; C → p,q,t; D → p]

Part – II (MATHEMATICS)

SECTION – I (Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

20. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Ans. **[C]**

Sol. Event (1) : original signal

OG : Original signal is green

OR : Original signal is red

Event (2) : Signal received by A.

AG : A received green

AR : A received Red

Event (3) : Signal received by B

BG : B received green

BR : B received Red

$$P\left(\frac{OG}{BG}\right) = \frac{P(OG).P\left(\frac{BG}{OG}\right)}{P(OG).P\left(\frac{BG}{OG}\right) + P(OR).P\left(\frac{BG}{OR}\right)}$$

$$= \frac{\frac{4}{5}\left[\frac{3}{4}.\frac{3}{4}+\frac{1}{4}.\frac{1}{4}\right]}{\frac{4}{5}\left[\frac{3}{4}.\frac{3}{4}+\frac{1}{4}.\frac{1}{4}\right]+\frac{1}{5}\left[\frac{1}{4}.\frac{3}{4}+\frac{3}{4}.\frac{1}{4}\right]} = \frac{20}{23}$$

21. If the distance of the point P (1, –2, 1) from the plane x + 2y –2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ (C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

Ans. **[A]**

Sol. $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\lambda$

foot $(1 + \lambda, -2 + 2\lambda, 1 - 2\lambda)$

$(1 + \lambda) + 2(-2 + 2\lambda) - 2(1 - 2\lambda) = 10$

$1 + \lambda - 4 + 4\lambda - 2 + 4\lambda = 10$

$9\lambda = 15$, $\Rightarrow \lambda = 5/3$

foot = $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$

$\frac{|1-4-2-\alpha|}{3}=5$

$|\alpha + 5| = 15$

$\alpha = 10$ (correct), –20 (wrong)

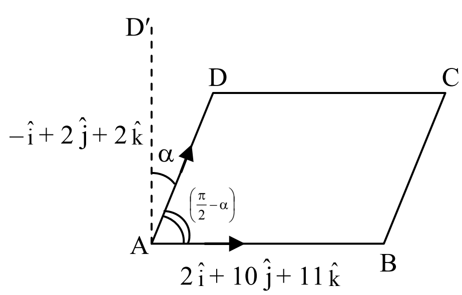
22. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Ans. **[B]**

Sol.



$\cos\left(\frac{\pi}{2}-\alpha\right) = \frac{\overrightarrow{AB}.\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} = \frac{40}{3(15)} = \frac{8}{9}$

$\sin\alpha = \frac{8}{9} \Rightarrow \cos\alpha = \frac{\sqrt{17}}{9}$

23. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25 (B) 34 (C) 42 (D) 41

Ans. **[D]**

Sol. S = {1, 2, 3, 4}

Possible subsets	No. of elements in Set A	Set B	Ways
	0	0	= 1
	1	0	$= {}^4C_1 = 4$
	2	0	$= {}^4C_2 = 6$
	1	1	$= {}^4C_2 = 6$
	3	0	$= {}^4C_3 = 4$
	2	1	$= {}^4C_2 .\, {}^2C_1 = 12$
	4	0	$= {}^4C_4 = 1$
	3	1	$= \frac{4!}{3!\ 1!} = 4$
	2	2	$= \frac{4!}{2!\ 2!\ 2!} = 3$

Total ⇒ 1 + 4 + 6 + 6 + 4 + 12 + 1 + 4 + 3 = 41

24. Let f be a real-valued function defined on the interval (–1, 1) such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all $x \in (-1, 1)$, and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{e}$

Ans. **[B]**

Sol. $f(x) = 2e^x + e^x \int_0^x \sqrt{t^4+1}\,dt$

$x = 0$; $f(0) = 2$

$f'(x) = 2e^x + e^x \int_0^x \sqrt{t^4+1}\,dt + e^x \sqrt{1+x^4}$

$f'(0) = 2 + 1 = 3$ $(f^{-1})'(2) = \frac{1}{f'(0)} = \frac{1}{3}$

25. For r = 0, 1, …, 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1+ x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

(A) $B_{10} - C_{10}$ (B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$ (C) 0 (D) $C_{10} - B_{10}$

Ans. **[D]**

Sol. $A_r = {}^{10}C_r$, $B_r = {}^{20}C_r$, $C_r = {}^{30}C_r$

$\sum_{r=1}^{10} {}^{10}C_r\ ({}^{20}C_{10}\ {}^{20}C_r - {}^{30}C_{10}\ {}^{10}C_r)$

$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_r\ {}^{20}C_{20-r} - {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_r . {}^{10}C_r$

$= {}^{20}C_{10}\ [{}^{30}C_{20} - {}^{10}C_0\ {}^{20}C_{20}] - {}^{30}C_{10}\ [{}^{20}C_{10} - ({}^{10}C_0)^2]$

$= {}^{20}C_{10}{}^{30}C_{20} - {}^{20}C_{10} - {}^{30}C_{10}\ {}^{20}C_{10} + {}^{30}C_{10}$

$= {}^{30}C_{10} - {}^{20}C_{10}$

$= C_{10} - B_{10}$

SECTION – II (Integer type)

This section contains **5 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question no. in the ORS is to be bubbled.

26. Let $a_1, a_2, a_3, \ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, …., 11.

If $\frac{a_1^2 + a_2^2 + \ldots.. + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots.. + a_{11}}{11}$ is equal to

Ans. **[0]**

Sol. $\because a_{k-1} = \frac{a_k + a_{k-2}}{2}$

so $\frac{a_1^2 + a_2^2 + \ldots.. + a_{11}^2}{11} = 90$

$\Rightarrow \Sigma(a + (r-1)\,d)^2 = 11 \times 90$

$\Rightarrow \Sigma(a^2 + 2ad\,(r-1) + (r-1)^2d^2) = 11 \times 90$

$11a^2 + 2ad\ \dfrac{10\times11}{2} + \dfrac{10\times11\times21}{6}\,d^2 = 11 \times 90$

so on solving d = –3

so $\dfrac{a_1 + a_2 + ... + a_{11}}{11}$

$= \dfrac{11}{2} . \dfrac{1}{11} . (2 \times a_1 + (11 - 1)(-3))$

$= \dfrac{1}{2}(30 - 30) = 0$

27. Let f be a function defined on R (the set of all real numbers) such that

$f'(x) = 2010\,(x - 2009)\,(x - 2010)^2\,(x - 2011)^3\,(x - 2012)^4$, for all $x \in R$.

If g is a function defined on R with values in the interval $(0, \infty)$ such that

$f(x) = \ell n\,\{g(x)\}$, for all $x \in R$, then the number of points in R at which g has a local maximum is

Ans. **[1]**

Sol. $g(x) = e^{f(x)}$

$g'(x) = e^{f(x)}\, f'(x)$

$g'(x) = 0 \Rightarrow f'(x) = 0 \Rightarrow x = 2009, 2010, 2011, 2012$

Points of local maxima = 2009, $\Rightarrow$ only one point

28. Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det(adj B) = 10^6, then [k] is equal to

[Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k]

Ans. **[4]**

Sol. $\det(A) = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$

$= (2k - 1)\,[-1 + 4k^2] - 2\sqrt{k}\,[-2\sqrt{k} - 4k\sqrt{k}\,] + 2\sqrt{k}\,[4k\sqrt{k} + 2\sqrt{k}\,]$

det (A) = (2k –1) (4k^2 –1) + 4 k (2k + 1) + 4k (2k +1)

= (2k –1) (4k^2 –1) + 8k (2k + 1)

det (B) = 0

det (adj A) = (det A)^2 = 10^6 det A = 10^3

$8k^3 + 1 - 2k - 4k^2 + 16k^2 + 8k = 10^3$

$8k^3 + 12k^2 + 6k - 999 = 0$

$k = 2 \rightarrow 64 + 48 + 12 - 999 < 0$

$k = 3 \rightarrow 8(27) + 109 + 18 - 999 < 0$

$k = 4 \rightarrow 8(64) + 12\,(16) + 24 - 999$

$512 + 192 + 24 - 999 < 0$

$k = 5 \rightarrow 8(125) + 12\,(25) + 6(5) - 999 > 0$

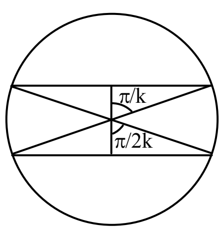
so [k] = 4

29. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is

[Note : [k] denotes the largest integer less than or equal to k]

Ans. **[3]**

Sol.



$$d = 2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k}$$

$$\frac{\sqrt{3}+1}{4} = \cos\frac{3\pi}{4k}\cos\frac{\pi}{4k} \Rightarrow \cos\frac{\pi}{4k}\cos\frac{3\pi}{4k} = \frac{\sqrt{3}+1}{2\sqrt{2}}\cdot\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{4k} = \frac{\pi}{12}$$

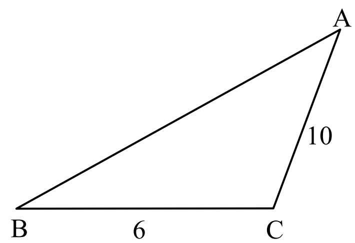
4k = 12

k = 3

30. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the in-circle of the triangle, then r^2 is equal to

Ans. **[3]**

Sol.



$\Delta = \frac{1}{2}$ ab sin C

$15\sqrt{3} = \frac{1}{2}6(10)\sin C \Rightarrow \sin C = \sqrt{3}/2$

$\Rightarrow C = 120°$

$\cos C = \frac{100+36-c^2}{2.10.6} \Rightarrow C^2 = 136 + 120° (1/2)$

$\Rightarrow C^2 = 196 \Rightarrow C = 14$

s = 15

$r = \frac{\Delta}{s} = \sqrt{3}$

$r^2 = 3$

SECTION – III (Paragraph Type)

This section contains **2 paragraphs**. Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 31 to 33

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of f(x) and let t = |s|.

31. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$ (C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

Ans. **[C]**

Sol. $f(x) = 4x^3 + 3x^2 + 2x + 1$

$f'(x) = 12x^2 + 6x + 2$ is always positive

$f(0) = 1$, $f(-1/2) = 1/4$, $f(-3/4) = -\frac{1}{2}$

so root $\in \left(\frac{-3}{4}, \frac{-1}{2}\right)$ $\because$ the equation have only one real root so s $\in \left(\frac{-3}{4}, \frac{-1}{2}\right)$ and t $\in \left(\frac{1}{2}, \frac{3}{4}\right)$

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$ (C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

Ans. **[A]**

Sol. $A(t) = \int_0^t f(x)\,d(x) = t^4 + t^3 + t^2 + t = t\left(\frac{1-t^4}{1-t}\right)$

$A(1/2) = 15/16$ & $A(3/4) = 3\left(\frac{175}{256}\right)$

So $A(t) \in \left(\frac{3}{4}, 3\right)$

33. The function $f'(x)$ is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in (–t, t)

(D) decreasing in (–t, t)

Ans. **[B]**

Sol. $f''(x) = 6(4x + 1)$

Paragraph for questions 34 to 36.

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4} = 1$, touching the ellipse at points A and B.

34. The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Ans. **[D]**

Sol. Equation of tangent

$y = mx \pm \sqrt{9m^2+4}$

as it passes through (3, 4)

so $4 = 3m \pm \sqrt{9m^2+4}$

$m = \frac{1}{2}$ and undefined.

So equation of the tangents will be

$x - 2y + 5 = 0$ and $x = 3$

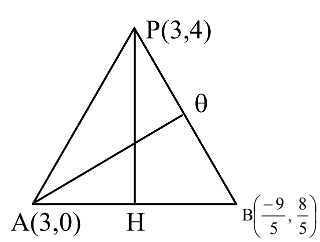
so point of contacts are (3, 0) and $\left(\frac{-9}{5}, \frac{8}{5}\right)$

35. The orthocentre of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Ans. **[C]**

Sol.



Equation of two altitudes PH and AQ are

$3x - y - 5 = 0$ and $2x + y - 6 = 0$ respectively

so orthocentre will be $\left(\frac{11}{5}, \frac{8}{5}\right)$

36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Ans. **[A]**

Sol. Equation of AB is $x + 3y - 3 = 0$

so required locus will be $(x - 3)^2 + (y - 4)^2 = \frac{(x+3y-3)^2}{10}$

$\Rightarrow 9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

SECTION – IV

(Matrix Type)

This section contains **2 questions.** Each question has **four statements** (A, B, C and D)given in **Column I** and five statements (p, q, r, s and t) in **column-II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column-II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

37. Match the statements in **Column-I** with those in **Column-II**.

[Note : Here z takes values in the complex plane and lm z and Re z denote, respectively, the imaginary part and the real part of z]

COLUMN-I	**COLUMN-II**
(A) The set of points z satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$ is contained in or equal to	(p) an ellipse with eccentricity $\frac{4}{5}$
	(q) the set of points z satisfying Im z = 0

(B) The set of points z satisfying

$|z + 4| + |z - 4| = 10$

is contained in or equal to

(r) the set of points z satisfying $|\text{Im } z| \le 1$

(C) If $|w| = 2$, then the set of points

$z = w - \frac{1}{w}$ is contained in or equal to

(s) the set of points z satisfying $|\text{Re } z| \le 2$

(D) If $|w| = 1$, then the set of points

$z = w + \frac{1}{w}$ is contained in or equal to

(t) the set of points z satisfying $|z| \le 3$

Ans. **A → q, r ; B → p ; C → p, s, t ; D → q, r, s, t**

Sol.

(A) Let $|Z| = r \ \forall \ r \in R$

$\left|\frac{Z - ir}{Z + ir}\right| = 1$ Which is the equation of line of perpendicular

bisector of $y = r$ & $y = -r$ that is $y = 0$

(B) $|Z + 4| + |Z - 4| = 10$

it will represent on ellipse

having foci (–4, 0), (4, 0)

so its equation will be $\frac{x^2}{25} + \frac{y^2}{9} = 1$

whose eccentricity is 4/5

(C) Let $w = 2e^{i\theta}$.

$z = \frac{3}{2}\cos + \frac{5}{2} i \sin\theta$

(D) Let $w = e^{i\theta}$

$Z = e^{i\theta} + e^{-i\theta}$

$= 2\cos\theta$.

38. Match the statements in **Column-I** with the values in **Column-II**.

COLUMN-I	**COLUMN-II**
(A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p) –4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are,	(q) 0
(C) Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b}=0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(D) Let f be the function on $[-\pi,\pi]$ given by $f(0)=9$ and $f(x)=\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x\neq 0$	(s) 5
The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(t) 6

Ans. **A → t ; B → p, r ; C → q, s ; D → r**

Sol.

(A) Let $P \equiv (\lambda + 2, 1 - 2\lambda, \lambda - 1)$

$Q \equiv (2\mu + \frac{2}{3}; -\mu - 3, \mu + 1)$

equation line PQ

$$\vec{r} = (\lambda + 2)\,\hat{i} + (1 - 2\lambda)\,\hat{j} + (\lambda - 1)\,\hat{k}$$

$$+ \alpha\left((2\mu - \lambda + \frac{2}{3})\,\hat{i} + (2\lambda - \mu - 4)\,\hat{j} + (\mu + 2 - \lambda)\,\hat{k}\right)$$

∴ This line passing through origin so.

$\lambda + 2 + \alpha\,(2\mu - \lambda + \frac{2}{3}) = 0$

$1 - 2\lambda + \alpha(2\lambda - \mu - 4) = 0$

$\lambda - 1 + \alpha(\mu - \lambda + 2) = 0$

on solving above three $\mu = \frac{1}{3}$ & $\lambda = 3$

So $P \equiv (5, -5, 2)$ & $Q \equiv (\frac{10}{3}, \frac{-10}{3}, \frac{4}{3})$

So $PQ = \sqrt{6} \Rightarrow (PQ)^2 = 6$

(B) $\tan^{-1}(x + 3) - \tan^{-1}(x - 3) = \sin^{-1}\frac{3}{5}$

$\tan^{-1}\frac{6}{x^2 - 8} = \tan^{-1}\frac{3}{4}$

$\Rightarrow\ x^2 - 8 = 8$

$\Rightarrow\ x^2 = 16 \Rightarrow x = \pm 4$

(C) $|\vec{b}|^2 + \vec{b}\,.\,\vec{c} = \vec{a}\,.\,\vec{c}$ …. (1)

put $\vec{a} = \mu\vec{b} + 4\vec{c}\ \ \forall\ \vec{a}\,.\,\vec{b} = 0 \Rightarrow \vec{b}\,.\,\vec{c} = -\frac{\mu}{4}|\vec{b}|^2$ …(2)

from (1) and (2)

$\frac{b^2}{c^2} = \frac{16}{4 - \mu + \mu^2}$ … (3)

$\therefore\ 2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$ and $\vec{a} = \mu\vec{b} + 4\vec{c}$

$\frac{b^2}{c^2} = \frac{12}{3 - 2\mu + \mu^2}$ … (4)

from (3) and (4)

m = 0,5

(D) $f(x) = \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}} = \frac{\sin 5x}{\sin x} + \frac{\sin 4x}{\sin x}$

$$I = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{4}{\pi}\int_{0}^{\pi} f(x)\,dx$$

$$= \frac{4}{\pi}\int_{0}^{\pi} \frac{\sin 5x}{\sin x}$$

$$= \frac{8}{\pi}\int_{0}^{\pi/2} \frac{\sin 5x}{\sin x}\,dx$$

$$= \frac{8}{\pi}\int_{0}^{\pi/2} \frac{\sin(3x+2x)}{\sin x}\,dx = \frac{8}{\pi}\int_{0}^{\pi/2} (1+2\cos 4x)\,dx$$

$$= 4$$

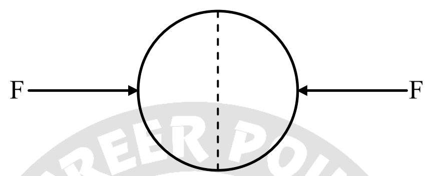
Part – III (PHYSICS)

SECTION – I

Single Correct Choice Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

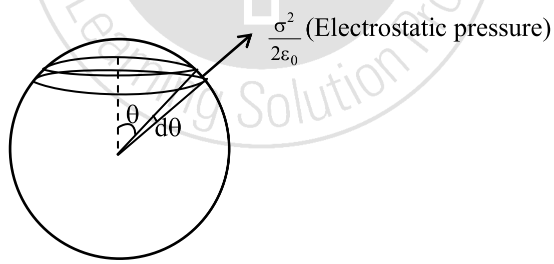
39. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to -



(A) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2 R$ (C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

Ans. **[A]**

Sol.



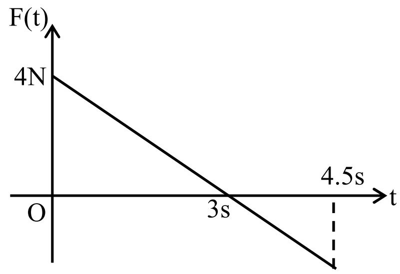
$$dA = 2\pi R \sin\theta \times R d\theta$$

$$dF = \frac{\sigma^2}{2\varepsilon_0} \times dA$$

Component of dF along vertical axis = dF cos θ

$$\text{Total force} = \int_0^{\pi/2} \frac{\sigma^2}{2\varepsilon_0}\pi R^2 \sin 2\theta d\theta = \frac{\sigma^2}{2\varepsilon_0} \times \pi R^2$$

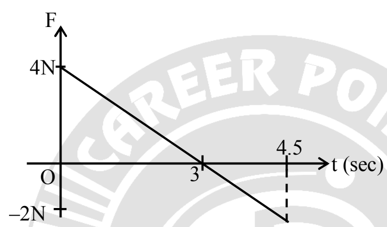
40. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time dependent force F(t) in the x direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is -



(A) 4.50 J (B) 7.50 J (C) 5.06 J (D) 14.06 J

Ans. **[C]**

Sol.



$m = -\frac{4}{3}$

At t = 4.5 sec $\vec{F} = -2N$

Total Impulse $I = \left[\frac{1}{2}\times 3\times 4\right] - \left[\frac{1}{2}\times 2\times 1.5\right]$

⇒ $= 6 - 1.5 = 4.5$ SI unit

Impulse = change in momentum

$4.5 = 2[v - 0]$

$v = \frac{4.5}{2} = 2.25$ m/sec

K.E. $= \frac{1}{2} \times 2 \times (2.25)^2 = 5.06$ J

41. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5\ Vm^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ ms^{-1}$. Given g = 9.8 m s^{-2}, viscosity of the air = $1.8 \times 10^{-5}\ Nsm^{-2}$ and the density of oil = 900 kg m^{-3}, the magnitude of q is -

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C (C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

Ans. **[D]**

Sol. $qE = mg$

$$\Rightarrow q\left[\frac{81\pi}{7}\times10^5\right] = 900 \times \frac{4}{3}\pi r^3 \times 9.8$$

$$q = \frac{900\times4\times r^3\times9.8\times7}{3\times81\times10^5} \quad \text{....... (1)}$$

$$v_T = 2 \times 10^{-3}\ \text{m/sec}$$

$$2 \times 10^{-3} = \frac{2}{9} \times \frac{r^2\times900\times9.8}{1.8\times10^{-5}}$$

$$r^2 = \frac{18\times1.8\times10^{-5}\times10^{-3}}{2\times900\times9.8} = 0.1836 \times 10^{-10} = 18.36 \times 10^{-12}$$

$$r = 4.284 \times 10^{-6}\ \text{m}$$

$$q = \frac{3600\times9.8\times7}{243\times10^5} \times 78.62 \times 10^{-18}$$

$$q = 0.799 \times 10^{-18} \approx 8 \times 10^{-19}\ \text{C}$$

42. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is -

(A) 5 grams (B) 10 grams (C) 20 grams (D) 40 grams

Ans. **[B]**

Sol. Fundamental frequency of closed pipe = $\frac{v}{4\ell}$

$\Rightarrow \frac{320}{4 \times 0.8} = \frac{320}{3.2} = 100$ Hz

Frequency of 2nd Harmonic of string = $\frac{v}{\ell} = \frac{1}{\ell}\sqrt{\frac{T}{\mu}}$

$100 = \frac{1}{\ell}\sqrt{\frac{50}{m/\ell}}$

$\Rightarrow 100 = \sqrt{\frac{50}{m \times 0.5}} \Rightarrow 100 = \sqrt{\frac{100}{m}}$

$10000 = \frac{100}{m} \Rightarrow m = 10^{-2}$ kg = 10 gm

43. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is -

(A) virtual and at a distance of 16 cm from the mirror

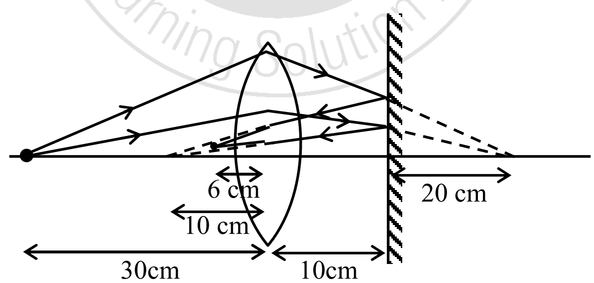
(B) real and at a distance of 16 cm from the mirror

(C) virtual and at a distance of 20 cm from the mirror

(D) real and at a distance of 20 cm from the mirror

Ans. **[B]**

Sol.



Refraction of reflected light by lens

f = + 15 cm

u = + 10 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{10} = \frac{1}{15}$

$v = 6$ cm

as incident rays are converging so refracted rays will converge more and final image is real.

44. A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier calipers, the least count is -

(A) 0.02 mm (B) 0.05 mm (C) 0.1 mm (D) 0.2 mm

Ans. **[D]**

Sol. Least Count = M.S. Reading – V.S. Reading .... (1)

20 V.S. = 16 M.S. or 16 mm

$1 \text{ V.S.} = \frac{16}{20}$ M.S. or $\frac{16}{20}$ mm

In equation (1)

Least Count = $\left(1 - \frac{16}{20}\right)$mm

= 0.2 mm

SECTION – II

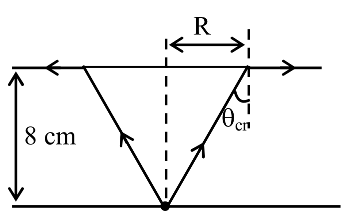
Integer Type

This section contains **Five questions.** The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

45. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R ?

Sol. **[6]**

$\sin \theta_{cr} = \frac{3}{5} \quad \Rightarrow \quad \tan \theta_{cr} = \frac{3}{4}$



$R = 8 \tan \theta_{cr}$

$= 8 \times \frac{3}{4} = 6$ cm

46. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour ?

Ans. **[3]**

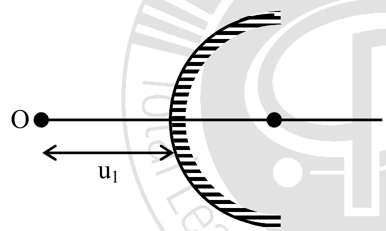
Sol. For position of object initially when image was at $\frac{25}{3}$ m

$$-\frac{1}{10} = -\frac{3}{25} + \frac{1}{u}$$

$$\frac{3}{25} - \frac{1}{10} = \frac{1}{u}$$

$$\frac{12-10}{100} = \frac{1}{u}$$

$u_1 = 50$

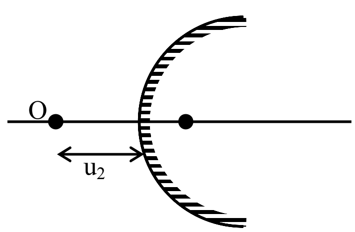


For position of object when image is at $\frac{50}{7}$ m

$$-\frac{1}{10} = -\frac{7}{50} + \frac{1}{u}$$

$$\frac{7}{50} - \frac{1}{10} = \frac{1}{u}$$

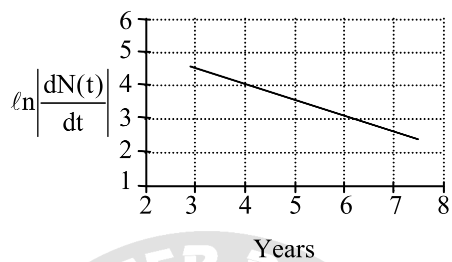
$u_2 = 25$



Speed of object = $\frac{50-25}{30} = \frac{25}{30}$ m/sec

$= \frac{25}{30} \times \frac{3600}{1000} = 3$ km/hr

47. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is -



Ans. **8**

Sol. From graph slope = $\frac{1}{2}$ = 0.5 year^{-1}

$\frac{dN}{dt} = Ne^{-\lambda t}$

$\ell n\left(\frac{dN}{dt}\right) = \ell n\,(N) - \lambda t$

so comparing we get λ = 0.5 year^{-1}

$t_{1/2} = \frac{0.693}{0.5}$ year

t = 4.16 years

so No. of half lives = $\frac{4.16}{0.693} \times 0.5 = 3$

$N_0 \rightarrow \frac{N_0}{2} \rightarrow \frac{N_0}{4} \rightarrow \frac{N_0}{8} \Rightarrow p = 8$

48. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. In the initial temperature of the gas is

T_i (in Kelvin) and the final temperature is aT_i, the value of a is -

Ans. **[4]**

Sol. for adiabatic process

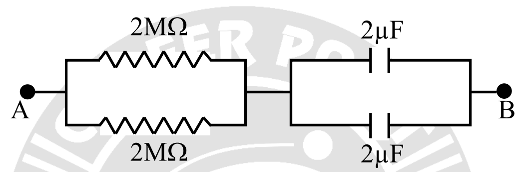
$TV^{\gamma-1}$ = const.

$$\Rightarrow T_i\ V^{\frac{7}{5}-1} = aT_i\left(\frac{V}{32}\right)^{\frac{7}{5}-1}$$

$\Rightarrow a = 4$

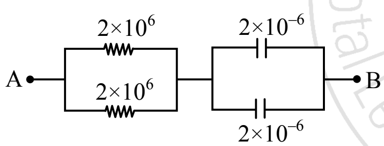
49. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V ?

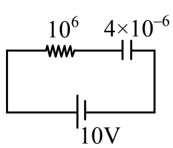
(Take : ℓn 5 = 1.6, ℓn 3 = 1.1]



Ans. **[2]**

Sol.





$q = CV_0\,(1 - e^{-t/RC})$

$V = V_0(1 - e^{-t/RC})$

$4 = 10(1 - e^{-t/4})$

$3 = 5e^{-t/4}$

$\log 3 = \log 5 - \frac{t}{4}$

$1.1 - 1.6 = -\frac{t}{4} \Rightarrow t = 2$ sec

SECTION – III

Comprehension Type

This section contains 2 paragraphs.. Based upon each of paragraph 3 multiple choice question have to be answered. Each of these questions has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Questions 50 to 52

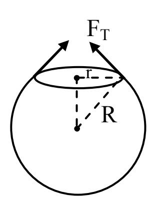
When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r << R) is

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

Ans. **[C]**

Sol.



$F_T = 2\pi rT$

Net vertically upward force

$\Rightarrow 2\pi rT\left(\frac{r}{R}\right) = \frac{2\pi r^2 T}{R}$

51. If r = 5× 10^{-4} m, ρ = 10^3 kgm^{-3}, g = $10ms^{-2}$, T = 0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

Ans. **[A]**

Sol. $\frac{2\pi r^2 T}{R} = \frac{4}{3}\pi R^3 \times \rho \times g$

$\Rightarrow \frac{2\times 25\times 10^{-8}\times 0.11}{R} = \frac{4}{3}\times R^3 \times 10^3 \times 10$

$\Rightarrow R^4 = \frac{50\times 3\times 0.11\times 10^{-8}}{4\times 10^4}$

$\Rightarrow R^4 = 4.125 \times 10^{-12}$

$\Rightarrow R = 1.4 \times 10^{-3}$ m

52. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

Ans. **[B]**

Sol. Surface energy = T(A) = $T \times 4\pi R^2$

$\Rightarrow 0.11 \times 4 \times 3.14 \times 1.96 \times 10^{-6} \quad \Rightarrow 2.7 \times 10^{-6}$ J

Paragraph for Questions 53 to 55

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ (C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

Ans. **[D]**

Sol. Bohr quantization principle

$$L = \frac{nh}{2\pi} = I\omega \Rightarrow \omega = \frac{nh}{2\pi I}$$

$$\text{Rotational KE} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{nh}{2\pi I}\right)^2 = \frac{n^2h^2}{8\pi^2 I}$$

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO moelcule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2

(C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

Ans. **[B]**

Sol. $\Delta E = E_2 - E_1$

$$= \frac{2^2h^2}{8\pi^2 I} - \frac{1^2h^2}{8\pi^2 I} = \frac{3h^2}{8\pi^2 I} = h\nu$$

When $\nu = \frac{4}{\pi}\times 10^{11}$ Hz

Solving $I = 1.87 \times 10^{-46}$ kg – m^2

55. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.) where 1 a.m.u. = $\frac{5}{3}\times 10^{-27}$ kg, is close to

(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m (C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

Ans. **[C]**

Sol. $I = m_1r_1^2 + m_2r_2^2$

Where m_1 = 12 amu

m_2 = 16 amu

$m_1r_1 = m_2r_2$

$r_1 + r_2 = r$ where r $\rightarrow$ distance between C & O.

Putting and solving

$r = 1.279 \times 10^{-10}$ m

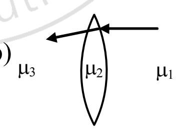
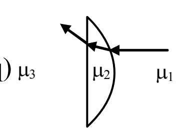
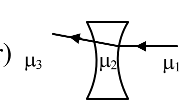
$\simeq 1.3 \times 10^{-10}$ m

SECTION – IV

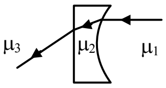
Matrix Type

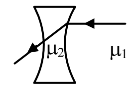
This Section contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

56. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **Column II**. A ray traversing these media is also shown in the figures. In **Column** I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column II**.

Column I	**Column II**
(A) $\mu_1 < \mu_2$	(p) 
(B) $\mu_1 > \mu_2$	(q) 
(C) $\mu_2 = \mu_3$	(r) 

(D) $\mu_2 > \mu_3$

(s) 

(t) 

Ans. **[A → p,r; B → q,s,t; C → p,r,t D → q,s]**

Sol. For (p) $\mu_2 > \mu_1$

as light rays bend towards normal at first refraction

$\mu_2 = \mu_3$ as no refraction occurs at second refraction

Option : (A), (C)

For (q)

$\mu_2 < \mu_1$ as bend away from normal at first refraction

$\mu_3 < \mu_2$ as bends away from normal at second refraction

Option (B), (D)

For (r)

$\mu_2 > \mu_1$ as bend towards the normal at first refraction

$\mu_2 = \mu_3$ as no refraction occurs at second refraction

Option (A), (C)

For (s)

$\mu_2 < \mu_1$ as bend away from normal at first refraction

$\mu_3 < \mu_2$ as bend away from normal at second refraction

Option (B), (D)

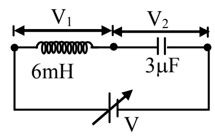
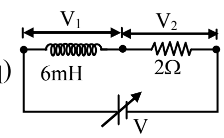
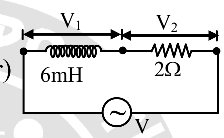
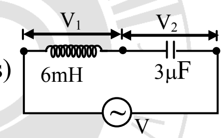
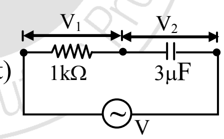
For (t)

$\mu_2 < \mu_1$ as bend away from normal at first refraction

$\mu_2 = \mu_3$ as no refraction occurs at second refraction

Option (B), (C)

57. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in **Column I**. Match the two

Column I	**Column II**
(A) $I \neq 0$, V_1 is proportional to I	(p) 
(B) $I \neq 0$, $V_2 > V_1$	(q) 
(C) $V_1 = 0$, $V_2 = V$	(r) 
(D) $I \neq 0$, V_2 is proportional to I	(s) 
	(t) 

Ans. **[A → r,s,t; B → q,r,s,t; C → p,q; D → q,r,s,t]**

Sol. For (p) Insteady state when I = constant

$V_L = 0 = V_1$

So $V_2 = V$

Option (C)

For (q) $V_1 = 0$ again as I = constant

$V_2 = V$

Also $V_2 = IR \Rightarrow$ Propotional to I.

Option (B), (C), (D)

For (r) $X_L = \omega L = (100\,\pi)\, 6 \times 10^{-3} \simeq 1.88\ \Omega$

$R = 2\Omega$

$V_1 = I\,X_L$; $V_2 = IR$

So $V_2 > V_1$

$V_2 \propto I$

also $V_1 \propto I$ Option (A), (B), (D)

For (s) $V_1 = I\,X_L$

$V_2 = I\,X_C$ where $X_C = \dfrac{1}{\omega C} \simeq 1061\ \Omega$

again $V_1 \propto I$; $V_2 \propto I$, $I \neq 0$

Option (A), (B) (D)

For (t) $V_1 = IR$ when $R = 1000\ \Omega$

$V_2 = I\,X_C$ when $X_C \simeq 1061\ \Omega$

$V_2 > V_1$

$V_1, V_2 \propto I$ and $I \neq 0$

Option (A), (B), (D)