PART - A

SUB-PART A-1

Q.1 The quantity \((\frac{nh}{2\pi qB})^{1/2}\) where \(n\) is a positive integer, \(h\) is Planck's constant, \(q\) is charge and \(B\) is magnetic field, has the dimensions of -
(A) area  
(B) speed  
(C) length  
(D) acceleration  
Sol. [C]
Consider the expression for the energy of a quantum \(E = hv\) to get the dimensions of \(h\) as those of (energy X time). Again the expression for the force \(F\) acting on a charge \(q\) moving with velocity \(v\) in a magnetic field \(B\) is \(qvB\) \(\sin\theta\). This gives the dimensions of \(qB\) as those of \((force/velocity)\). Use these to determine the dimensions of the quantity under consideration, nothing that \(n\) is a dimensionless quantity.

Q.2 In vernier calipers, \(m\) divisions of main scale coincide with \((m + 1)\) divisions of vernier scale. If each division of main scale is \(d\) units, the least count of instrument is -
(A) \(\frac{d}{m}\)  
(B) \(\frac{d}{m+1}\)  
(C) \(\frac{md}{m+1}\)  
Sol. [B]
Since \(m\) divisions of main scale are equivalent to \((m + 1)\) divisions of vernier scale, one division of vernier scale is equivalent to \([\frac{m}{m+1}]\) divisions of main scale. Now, use the definition : least count of vernier = one division of main scale – one division of vernier scale.

Q.3 Vectors \(a\) and \(b\) include an angle \(\theta\) between them. If \((a + b)\) and \((a – b)\) respectively subtend angles \(\alpha\) and \(\beta\) with \(a\), then \((\tan \alpha + \tan \beta)\) is
(A) \(\frac{ab \sin \theta}{a^2 + b^2 \cos^2 \theta}\)  
(B) \(\frac{(a^2 – b^2 \cos^2 \theta)}{(a^2 + b^2 \cos^2 \theta)}\)  
(C) \(\frac{(a^2 \sin^2 \theta)}{(a^2 + b^2 \cos^2 \theta)}\)  
(D) \(\frac{(b^2 \sin^2 \theta)}{(a^2 – b^2 \cos^2 \theta)}\)  
Sol. [B]
Use the relation : \(\tan \alpha = \frac{b \sin \theta}{a + b \cos \theta}\) where \(\alpha\) is the angle made by the vector \((a + b)\) with \(a\). Similarly, \(\tan \beta = \frac{b \sin \theta}{a – b \cos \theta}\) where \(\beta\) is the angle made by the vector \((a – b)\) with \(a\). Note that angle between \(a\) and \(–b\) is \((180^\circ - \theta)\).

Q.4 A small bob attached to a string of length \(l\) is suspended from a rigid support and rotates with uniform speed along a circle in a horizontal plane. Let \(\theta\) be the angle made by the string with the vertical. Then the length of a simple pendulum having the same period is -
(A) \(\frac{l}{\cos \theta}\)  
(B) \(\frac{l}{\sin \theta}\)  
(C) \(\frac{l}{\sin \theta}\)  
(D) \(\frac{l}{\cos \theta}\)  
Sol. [D]
The period of revolution of a conical pendulum is \(T = 2\pi \sqrt{\frac{l \cos \theta}{g}}\). This can be compared with the expression of period of a simple pendulum.

Q.5 Acceleration – time graph of a particle moving along \(X\) axis is as shown. The particle will have the velocity same as its initial velocity at –

\[
\begin{align*}
a & = m/s^2 \\
O & 5 \\
10 & S
\end{align*}
\]
(A) \(10 \ s\)  
(B) \((10 + \sqrt{3}) \ s\)  
(C) \((10 + 5\sqrt{3}) \ s\)  
(D) \((10 + 2\sqrt{3}) \ s\)  
Sol. [C]
Let \(A\) be the acceleration at \(t = 0\). Change in velocity in first 10 second will be \(5A + (1/2) 5A = 15A/2\) which is equal to the area under the curve. To have velocity same as the initial velocity, the net change in velocity should be zero. For this the area under the curve has to be \(\left(-\frac{15A}{2}\right)\). Note that the area below the \(X\) axis is negative. Slope of straight line after \(t = 5\)s is \((- A/5)\). If point \((10, 0)\) is used as origin, equation of the line will be given by \(y = (– A/5)x\). At \(x = t_1, y = (– A/5)T_1\). Now, area between \(x = 10\) and \(x = t_1\) will be \([(1/2)t_1 \times X = (– A/5)T_1] = (– t_1^2A)/10\). This can be equated to \((– 15A/2)\) so as to get \(t_1^2 = 75\) or \(t_1 = 5\sqrt{3}\)s. this is the time from \(10\) s and hence the total time is \((10 + 5\sqrt{3})\) second.
Q.6 A block of mass \( m \) is placed on an inclined plane with angle of inclination \( \theta \). Let \( N \), \( f_l \) and \( F \) respectively represent the normal reaction, limiting force of friction and the net force down the inclined plane. Let \( \mu \) be the coefficient of friction. The dependence of \( N \), \( f_l \) and \( F \) on \( \theta \) is indicated by plotting graphs as shown below. Then, curves (1), (2) and (3) respectively represent

(A) \( N \), \( F \) and \( f_l \)  
(B) \( F \), \( f_l \) and \( N \)  
(C) \( F \), \( N \) and \( f_l \)  
(D) \( f_l \), \( N \) and \( F \)

Sol.  
[C]  
The normal reaction \( N (= mg \cos \theta) \) and the frictional force \( f_l (= \mu mg \cos \theta) \) vary as cosine of the angle of inclination \( \theta \). The net downward force is given by \( [mg \sin \theta – \mu mg \cos \theta] \). Note that the downward force is zero up to the angle of repose, that is, till the frictional force has not attained its limiting value.

Q.7 When a solid sphere rolls without slipping down an inclined plane making an angle \( \theta \) with the horizontal, the acceleration of its centre of mass is \( a \). If the same sphere slides without friction its acceleration \( a' \) is

(A) \( (7/2) a \)  
(B) \( (5/7) a \)  
(C) \( (7/5) a \)  
(D) \( (5/2) a \)

Sol.  
[C]  
The acceleration of the sphere when it slides without friction is \( g \sin \theta \). When the sphere rolls without slipping, the acceleration is \( (5/7) (g \sin \theta) \) and hence the answer.

Q.8 A 2 kg cylinder rests on a cart as shown in the figure. The coefficient of static friction between the cylinder and the cart is 0.5. The cylinder is 4 cm in diameter and 10 cm in height. Taking \( g = 10 \, m/s^2 \), the minimum acceleration of the cart needed to make the cylinder tip over is about

(A) 2 ms\(^{-2}\)  
(B) 4 ms\(^{-2}\)  
(C) 8 ms\(^{-2}\)  
(D) 6 ms\(^{-2}\)

Sol.  
[B]  
Consider the cart moving to the right with acceleration \( a \). Then, a pseudo force \( ma \) acts on the cylinder (of mass \( m \)) to the left. In the limiting case, taking the moments about a point of contact of the cylinder with the cart on the left, clockwise moment \( (ma \times h/2) \) = anticlockwise moment \( (ma \times d/2) \) where \( h \) and \( d \) are the height and diameter of the cylinder respectively.

Q.9 Water flows out of the hole on the side of a bucket and follows a parabolic path. If the bucket falls freely under gravity, ignoring air resistance, the water flow -

(A) follows a straight line path relative the falling bucket  
(B) follows a parabolic path relative the falling bucket  
(C) decreases but continues to flow  
(D) stops

Sol.  
[D]  
Under a free fall the water head above the hole does not exert any pressure. Then, the pressures on the two sides of the hole become equal and hence the flow stops.

Q.10 A spring has length \( l \) and spring constant \( k \). It is cut into two pieces of lengths \( l_1 \) and \( l_2 \) such that \( l_1 = n l_2 \). The force constant of spring of length \( l_1 \) is

(A) \( k (l + n) \)  
(B) \( k (l + n) / n \)  
(C) \( k \)  
(D) \( k (l + n) \)

Sol.  
[B]  
The length \( l_1 = [n/(n + 1)]l \). Noting that the elongation is proportional to the length, the force constant becomes \( [k (n + 1)/n] \).
Physic

Q.11 Consider two hollow glass spheres, one containing water and the other containing mercury. Each liquid fills about one tenth of the volume of the sphere. In zero gravity environment -
(A) water and mercury float freely inside the spheres
(B) water forms a layer on the glass while mercury floats
(C) mercury forms a layer on the glass while water floats
(D) water and mercury both form a layer on the glass

Sol. [B]
Note that cohesive force among mercury molecules is greater than adhesive force between glass and mercury molecules. Also, adhesive force between water and glass molecules is greater than cohesive force among water molecules.

Q.12 A uniformly thick plate in the shape of an arrowhead has dimensions as shown. The centre of mass lies at a point

6 cm
3 cm
6 cm

(A) 1.5 cm to the right of O
(B) 3 cm to the right of O
(C) O itself
(D) 1 cm to the right of O

Sol. [D]
Note that the centre of mass of a uniform thick triangular sheet is at the centroid which divides a median in a ratio 2 : 1. The required center of mass must be on the line of symmetry passing through O and the vertex to the right. Consider the shape of arrowhead to be obtained by cutting the triangular part to the left (with base 6 cm and height 3 cm) from the uncut triangular sheet with base 6 cm and height 9 cm. The ratio of masses of these two is 27 : 9, the thickness being uniform.

Q.13 A disc of radius R = 10 cm oscillates as a physical pendulum about an axis perpendicular to the pane of the disc at a distance r from its center. If r = R/4, the approximate period of oscillation is -
(A) 0.84 s
(B) 0.94 s
(C) 1.26 s
(D) 1.42 s

Sol. [B]
Use the expression for the periodic time of a physical pendulum : \[ T = 2 \pi \sqrt{\frac{I}{mgh}} \]
where I is the moment of inertia of the disc about the axis under consideration. In this case it is \( (mR^2/2 + mR^2/16) = 9mR^2/16 \). The distance of the centre of mass from the point of suspension is \( h = R/4 \).

Q.14 Sachin (55 kg) and Kapil (65 kg) are sitting at the two ends of a boat at rest in still water. The boat weighs 100 kg and is 3.0 m long. Sachin walks down to Kapil and shakes hands. The boat gets displaced by -
(A) zero m
(B) 0.75 m
(C) 3.0 m
(D) 2.3 m

Sol. [B]
Kapil and the boat can be considered as one body of mass \( m_b = (65 +100) = 165 \) kg. Note that the centre of mass of the system remains unchanged since no external force acts on the system. Let \( m_s \) be the mass of Sachin and \( \Delta x_s, \Delta x_b \) be the displacements of the combined body of mass \( m_b \) and Sachin respectively with reference to the centre of mass. Then use the equation \( m_s \Delta x_s + m_b \Delta x_b = 0 \), to get the answer.

Q.15 A uniform solid disc of radius R and mass m is free to rotate on a frictionless pivot through a point on its rim. The disc is released from rest in the position where the diameter through the pivot is along horizontal. The speed of its centre of mass when the diameter through the pivot is vertical is
(A) \( (2/3)(gR)^{1/2} \)
(B) \( (gR)^{1/2} \)
(C) \( (2gR)^{1/2} \)
(D) \( 2(gR/3)^{1/2} \)

Sol. [D]
Moment of inertia about the axis through the pivot on the rim of the disc is \( (mR^2/2 + mR^2) = 3mR^2/2 \). using the principle that change in potential energy (mgR) is equal to the gain of kinetic energy \( (Io^2/2) \) gives the answer. Also use \( v = R \omega \).
Q.16 A 40.0 kg boy is standing on a plank of mass 160 kg. The plank originally at rest, is free to slide on a smooth frozen lake. The boy walks along the plank at a constant speed of 1.5 m/s relative to the plank. The speed of the boy relative to the ice surface is -
(A) + 1.8 m/s  
(B) – 1.2 m/s  
(C) + 1.2 m/s  
(D) + 1.5 m/s

Sol.  
(C)

The system is not subjected to any external force and hence conservation of momentum can be used. Let \( m_b \) and \( m_p \) represent the masses of the boy and the plank. Let \( v_{bi} \), \( v_{pi} \) and \( v_{bp} \) be the velocity of the boy with respect to ice, that of the plank with respect to ice and that of the boy with respect to the plank respectively. Then, \( m_b v_{bi} + m_p v_{pi} = 0 \), also \( v_{bi} = v_{bp} + v_{pi} \).

Q.17 When a soap bubble is given an electric charge,
(A) it contracts  
(B) it expands  
(C) its size remains the same  
(D) it expands or contracts depending upon whether the charge is positive or negative

Sol.  
(B)

For an unelectrified soap bubble force due to excess pressure from inside is balanced by the force due to surface tension. When it is given an electric charge, there is an outward normal force \( (\sigma_r^2/2\epsilon_0) \) per unit area, where \( \sigma \) is the surface charge density that expands the bubble.

Q.18 A wooden block floats in a liquid with 40\% of its volume inside the liquid. When the vessel containing the liquid starts rising upwards with acceleration \( a = g/2 \), the percentage of volume inside the liquid is -
(A) 20\%  
(B) 60\%  
(C) 30 \%  
(D) 40\%

Sol.  
(D)

When the vessel is stationary, the weight of the wooden block is balanced by the upthrust, that is \( V \rho_{wood} g = V_{liq} \rho_{liq} g \), where \( V \) is the volume of the block and \( V_{liq} \) is the volume of the liquid displaced, \( \rho_{wood} \) and \( \rho_{liq} \) are the densities of wood and liquid respectively. This gives \( V_{liq} / V = (\rho_{wood} / \rho_{liq}) \). When the vessel moves up, the net upward force is (upthrust – weight). The upthrust is \( (V_{liq} \rho_{liq} g) \) where \( V_{liq} \) is the volume of the liquid displaced in this case. The net upward force is \( V \rho_{wood} g/2 \). This gives \( V_{liq} / V = (\rho_{wood} / \rho_{liq}) \). From this we see that the same volume of the wooden block remains inside the liquid.

Q.19 A metal wire of length \( L_1 \) and area of cross section \( A \) is attached to a rigid support. Another metal wire of length \( L_2 \) and of the same cross sectional area is attached to the free end of the first wire. A body of mass \( M \) is then suspended from the free end of the second wire. If \( Y_1 \) and \( Y_2 \) are the Young's moduli of the wires respectively, the effective force constant of the system of two wires is -
(A) \( (Y_1 Y_2 A) / [2(Y_2 L_2 + Y_2 L_1)] \)  
(B) \( (Y_1 Y_2 A) / (L_1 L_2)^{1/2} \)  
(C) \( (Y_1 Y_2 A) / (Y_2 L_2 + Y_2 L_1) \)  
(D) \( (Y_1 Y_2) / (L_1 L_2)^{1/2} \)

Sol.  
(C)

Using the usual expression for the Young's modulus, the force constant for the wire can be written as \( k = F/\Delta l = YA/l \) where the symbols have their usual meanings. Now, the two wires together will have an effective force constant \( k(1/k_2 + 1/k_2) \). Substituting the corresponding lengths and the Young's moduli we get the answer.

Q.20 Four moles of carbon monoxide are mixed with four moles of carbon dioxide. Assuming the gases to be ideal, the ratio of specific heats is -
(A) 15/11  
(B) 41/30  
(C) 4/3  
(D) 7/4

Sol.  
[A]

Use the expression for the ratio of specific heats of a mixture:
\[ \gamma = \frac{n_1 C_p + n_2 C_v}{n_1 C_v + n_2 C_p} \]

Also note that \( C_p \) and \( C_v \) for diatomic gas are \( 5R/2 \) and \( 7R/2 \) respectively, whereas those for a polyatomic gas are \( 3R \) and \( 4R \).

Q.21 The equations of two sound waves propagating in a medium are given by \( y_1 = 2 \sin (200 \pi t) \) and \( y_2 = 5 \sin (150 \pi t) \). The ratio of intensities of sound produced is -
(A) 4 : 25  
(B) 9 : 100  
(C) 8 : 15  
(D) 64 : 225

Sol.  
[D]

The intensity of a wave is proportional to square of amplitude as well as square of frequency. The amplitudes are in the ratio 2 : 5 whereas the frequencies are in the ratio 4 : 3.
Q.22 String A has a length \( l \), radius of cross section \( r \), density of material \( \rho \) and is under tension \( T \). String B has all these quantities double those of string A. If \( f_A \) and \( f_B \) are the corresponding fundamental frequencies of the vibrating strings, then
(A) \( f_A = 2 f_B \)  
(B) \( f_A = 4 f_B \)  
(C) \( f_B = 4 f_A \)  
(D) \( f_A = f_B \)  

Sol.  [B]  
Note that the frequency of the vibrating string 
\[ f \propto \frac{T}{l \sqrt{A \times \rho}} \]  
where the symbols have their usual meanings.

Q.23 The temperature of \( n \) moles of an ideal gas is increased from \( T \) to \( 4T \) through a process for which pressure \( p = aT^{-1} \) where \( a \) is a constant. Then, the work done by the gas is
(A) \( nRT \)  
(B) \( 4nRT \)  
(C) \( 2nRT \)  
(D) \( 6nRT \)  

Sol.  [D]  
Use the relation \( pV = nRT \) with \( p = a/T \) (given). This gives \( V = nRT^2 / a \), so that \( dV = (2nRT / a)dT \). Now integrate \( pdV \) between \( T \) and \( 4T \) to get the result.

Q.24 For a monatomic ideal gas undergoing an adiabatic change, the relation between temperature and volume is \( TV^{\gamma} = \text{constant} \) where \( \gamma \) is -
(A) \( 7/5 \)  
(B) \( 2/5 \)  
(C) \( 2/3 \)  
(D) \( 1/3 \)  

Sol.  [C]  
For an adiabatic change in case of a monatomic gas, \( TV^{\gamma - 1} = \text{constant} \). In this case \( x \) itself is \( (\gamma - 1) \) and \( \gamma = 5/3 \) giving the value of \( x \).

Q.25 A system is taken from a given initial state to a given final state along various paths represented on a p-V diagram. The quantity that is independent of the path is
(A) amount of heat transferred \( Q \)  
(B) amount of work done \( W \)  
(C) \( Q \) but not \( W \)  
(D) \( Q - W \)  

Sol.  [D]  
The only quantity \( (Q - W) \) which itself is the internal energy of the system is independent of the path.

Q.26 An ideal gas confined to an insulated chamber is allowed to enter into an evacuated insulated chamber. If \( Q \), \( W \) and \( \Delta E_{\text{int}} \) have the usual meanings, then
(A) \( Q = 0, W \neq 0 \)  
(B) \( W = 0, Q \neq 0 \)  
(C) \( \Delta E_{\text{int}} = 0, Q \neq 0 \)  
(D) \( Q = W = \Delta E_{\text{int}} = 0 \)  

Sol.  [D]  
This is a case of free expansion of a gas. Note that due to insulation, \( Q = 0 \). Since the gas expands against no counteracting pressure (that is in vacuum), \( W = 0 \). This gives no change in the internal energy.

Q.27 Read the two statements – (I) When a solid melts an changes to liquid state, its volume may increase or decrease. (II) As a result of increase in pressure, the melting point of a solid may be raised or lowered.  
With reference to these statements, the only correct statements out of the following is
(A) (I) is true but (II) cannot be true  
(B) (I) cannot be true but (II) is true  
(C) (I) and (II) both are true and (I) is the cause of (II)  
(D) (I) and (II) both are true and they are independent of each other  

Sol.  [C]  
An increase in pressure tends to compress the substance. On melting if volume of a substance decreases, an increase in pressure will help the process of melting, so that melting point will be lower. On the other hand if volume of a substance increases on melting, then an increase in pressure will oppose the process of melting. hence, melting point will increase. Thus, the two statements are true and (I) is the cause of (II).

Q.28 A magnetic field directed along Z axis varies as \( B = B_0x/a \), where \( a \) is a positive constant. A square loop of side \( l \) and made of copper is placed with its edges parallel to X and Y axes. if the loop is made to move with a constant velocity \( v_0 \) directed along X axis, the emf induced is
(A) \( (B_0v_0)^2/a \)  
(B) \( B_0v_0l \)  
(C) \( (B_0v_0l)^2/2a \)  
(D) \( (B_0v_0l)^2/a^2 \)  

Sol.  [D]  
The only quantity \( (Q - W) \) which itself is the internal energy of the system is independent of the path.
Q.29 An object is placed in front of a spherical mirror of focal length f. If x and x' respectively represent the distances of the object and the image from the focus, then -
(A) f = x + x'
(B) f^2 = x x'
(C) f = |x - x'|
(D) f = x ± x' depending upon whether image is real or virtual
Sol. [B]
Note that the object distance can be written as (f + x) whereas the image distance can be written as (f + x^2). Use the mirror formula to get the answer.

Q.30 Different objects at different distance are seen by the eye. The parameter that remains constant is -
(A) the focal length of the eye lens
(B) the object distance from the eye lens
(C) the radii of curvature of the eye lens
(D) the image distance from the eye lens
Sol. [D]
The image formed by the eye lens is always one the retina and the image distance is fixed.

Q.31 A body in the form of a right circular cone of dielectric material with base radius R and height h is placed with its base on a horizontal table. A horizontal uniform electric field of magnitude E penetrates the cone. The electric flux that enters the body is
(A) ERh/3 (B) ERh (C) ERh/6 (D) 2ERh
Sol. [B]
Note that the flux through an area is (E . dA). Here the flux through the cone is the same as that through the triangular section of the cone by a vertical plane passing through the vertex. The area of this triangular section is [1/2(2R X h)] and is perpendicular to the direction of the field E.

Q.32 A sounding body emitting a frequency of 200 Hz is released from a height. One second after its release, it crosses a balloon rising with a constant velocity of 2m/s. Let the speed of sound be 300 m/s and acceleration due to gravity 10 m/s^2. The change in frequency noted by an observer in the balloon at the moment of crossing is -
(A) 10 Hz (B) 8 Hz (C) 16 Hz (D) 4 Hz
Sol. [C]
Use the relation : the apparent frequency
\[ n^2 = n \left( \frac{v + v_0}{v - v_s} \right) \]
where v is the speed of sound, v_0 is the speed of the observer and v_s is the speed of the source of sound. The speed v_0 is to be considered positive or negative depending on whether the observer is moving towards or away from the source. Similarly, v_s is to be considered positive or negative depending on whether the source is moving towards or away from the observer. Using this convention, determine the apparent frequencies before and after the crossing and then the difference between them.

Q.33 Refer to the figure. The number reflections from mirrors M_1 and M_2 are –
(A) 5 and 5  (B) 6 and 5
(C) 10 and 10 (D) 6 and 6
Sol. [B]
Consider the point at which the incident ray strikes M_1 for the first time. Let its distance from the lower end of the mirror be x, so that tan 5° = (x/1) giving x = 0.087 (approximately). If n is the number of spacings accommodated in the mirror length of 1 meter, then nx = 1, giving n = 11.49. Thus, n is greater than 11 but less than 12. This gives 6 reflections from M_1 but 5 from M_2.
Q.34 A ray of light is incident at angle $\alpha$ on the boundary separating two transparent media. It is transmitted. If the angle of incidence is increased very slightly, the ray gets reflected in the same medium. The difference between angles of deviation in the two cases will be close to -
(A) $2\alpha$  
(B) $90^\circ - \alpha$  
(C) $180^\circ - \alpha$  
(D) $180^\circ - 2\alpha$

Sol. [B]
When the ray is incident at the critical angle $\alpha$, the angle of deviation is $(90^\circ - \alpha)$ whereas just after this (that is at an angle slightly greater than the critical angle) the angle of deviation is $(180^\circ - 2\alpha)$

Q.35 Two particles A and B having equal charged after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii $R_1$ and $R_2$ respectively. The ratio of mass of A to that of B is
(A) $(R_1/R_2)^2$  
(B) $(R_1/R_2)^{1/2}$  
(C) $(R_2/R_1)^2$  
(D) $(R_2/R_1)$

Sol. [A]
The work done by the electric field on both the particles is the same and hence the gain in kinetic energy for both is the same. This gives $v_2^2 = \frac{m_1}{m_2} v_1^2$. Note that the centripetal force necessary for the circular motion in the magnetic field is $qvB$. Using this for both the particles, we get $\frac{m_1 v_1}{R_1} = \frac{m_2 v_2}{R_2}$. From these two relations, we get the answer.

Q.36 A Sound wave traveling through a medium of bulk modulus $B$ is represented as $y(x, t) = A \sin (kx - \omega t)$ where symbols have their usual meanings. Then, the corresponding pressure amplitude is
(A) $BAk$  
(B) $B (A/K)^{1/2}$  
(C) $B$  
(D) $B(AK)^{1/2}$

Sol. [A]
From the equation of the sound wave, we get $\frac{\partial y}{\partial x} = kA \cos (kx - \omega t)$. Now use the expression $\Delta p = -B \frac{\partial y}{\partial x}$ where $B$ is the bulk modulus. This gives an expression for the pressure change $\Delta p = -BAk \sin [(kx - \omega t) - \pi/2]$, also indicating a phase lag of $\pi/2$ with respect to displacement.

Q.37 A conductor is bent in the form of concentric semicircles as shown in the figure. The magnetic field at the point O is:

(A) zero  
(B) $\frac{\mu_0 i}{6a}$  
(C) $\frac{\mu_0 i}{a}$  
(D) $\frac{\mu_0 i}{4a}$

Sol. [B]
Magnetic field at the centre of a semicircular current carrying conductor is given by the expression $B = \frac{\mu_0 i (\pi a)}{4\pi a^2}$ where $a$ is the radius of the first semicircle. Note that the current in all the turns is the same but its sense is alternately opposite and the radii are in the proportion $1 : 2 : 4 : 8 \ldots \ldots$ Then, the net magnetic field = $\frac{\mu_0 i}{4a} \left[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \right]$. The terms in the bracket form a geometric progression which adds to $(2/3)$ and then the answer follows.

Q.38 A capacitor and a coil in series are connected to a 6 volt ac source. By varying the frequency of the source, maximum current of 600 mA is observed. If the same coil is now connected to a cell of emf 6 volt and internal resistance of 2 ohm, the current through it will be
(A) 0.5 A  
(B) 0.6 A  
(C) 1.0 A  
(D) 2.0 A

Sol. [A]
The maximum current is obtained at resonance where the net impedance is only resistive which is the resistance of the coil only. This gives the resistance of the coil as 10 ohm. Now, this coil along with the internal resistance of the cell gives a current of 0.5 A.

Q.39 Two radioactive substances X and Y emit $\alpha$ and $\beta$ particles respectively. Their disintegration constants are in the ratio 2 : 3. To have equal probabilities of getting emission of $\alpha$ and $\beta$ particles, the ratio of number of atoms of X to that of Y at any time instant is:
(A) 2 : 3  
(B) 3 : 2  
(C) $e : 1$  
(D) $(e - 1) : 1$

Sol. [B]
The maximum current is obtained at resonance where the net impedance is only resistive which is the resistance of the coil only. This gives the resistance of the coil as 10 ohm. Now, this coil along with the internal resistance of the cell gives a current of 0.5 A.
Note that \((\lambda_X/\lambda_Y) = 2/3\). To have equal probabilities the rates of decay must be equal, that is, \(\lambda_XN_X = \lambdaYN_Y\) at any instant. The gives the ratio \((N_X/N_Y) = 3/2\).

Q.40 In a certain particle accelerator, electrons emerge in pulses at the rate of 250 pulses per second. Each pulse is of duration of 200 ns and the electrons in the pulse constitute a current of 250 mA. The number of electrons delivered by the accelerator per pulse is:

(A) \(8.00 \times 10^{10}\)  
(B) \(5.00 \times 10^8\)  
(C) \(3.13 \times 10^{11}\)  
(D) \(9.60 \times 10^{10}\)

Sol.  
[C]

The charge in a pulse \(dq = I dt = 5.00 \times 10^{-8} \text{C}\).
Divide this by \(1.6 \times 10^{-19}\) to determine the number of electrons per pulse.

Q.41 Let \(v(t)\) be the velocity of a particle at time \(t\). Then,

(A) \(|dv(t)/dt|\) and \(d|v(t)|/dt\) are always equal
(B) \(|dv(t)/dt|\) and \(d|v(t)|/dt\) may be equal
(C) \(d|v(t)|/dt\) can be zero while \(|dv(t)/dt|\) is not zero
(D) \(d|v(t)|/dt\) ≠ 0 implies \(|dv(t)/dt|\) ≠ 0

Sol.  
[B, C, D]

Note that \(|dv(t)/dt|\) is the magnitude of acceleration, while \(d|v(t)|/dt\) is the time rate of change of speed. These two may not be always equal, hence (a) is not correct. In fact, the two are equal when the motion is along a straight line. In case of uniform circular motion, speed remains constant but not the velocity. Again if speed is not constant the velocity cannot be constant.

Q.42 A hollow double concave lens is made of a very thin transparent material. It can be filled with water (refractive index \(\mu_w\)) or either of two liquids \(L_1\) or \(L_2\) with refractive indices \(\mu_1\) and \(\mu_2\) respectively (\(\mu_2 > \mu_w > \mu_1\)). The lens will diverge a parallel beam of light incident on it, if it is filled with -

(A) \(L_2\) and immerses in \(L_1\)
(B) \(L_2\) and immersed in water
(C) water and immersed in \(L_1\)
(D) air and immersed in either water or \(L_1\) or \(L_2\)

Sol.  
[A, B, C]

Use the lens maker formula:

\[
\frac{1}{f} = \left(\frac{\mu_1}{\mu_m} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)
\]

where \(\mu_L\) and \(\mu_m\) represent the refractive indices of the liquid filled and the surrounding medium respectively. Note that for a double concave lens \[\frac{1}{R_1} - \frac{1}{R_2}\] is negative. Now, for the lens to act as a diverging lens, that is, for \(f\) to be negative, the first bracket on the right hand side of the relation must be positive. This requires \(\mu_L\) to be greater than \(\mu_m\).

Q.43 Referring to the circuit diagram, the tap key is pressed at time \(t = 0\). After sufficiently long time

(A) \(V_C = 0\)  
(B) \(V_R = 0\)  
(C) \(V_C = V\)  
(D) \(V_R = V\)

Sol.  
[B, C]

After sufficiently long time the charging current reduces to zero. Then all the voltage appears across the capacitor and no drop across the resistor.

Q.44 As shown in the figure, a front coated mirror \(M\) produces an image \(S_2\) of a source \(S_1\) of monochromatic light. Then,

(A) Point \(P\) will be a point of maximum intensity if the path difference \(\Delta = (2n)\lambda/2\) for \(n = 0, 1, 2, 3, \ldots\)
(B) Point \(P\) will be a point of maximum intensity if the path difference \(\Delta = (2n + 1)\lambda/2\) for \(n = 0, 1, 2, 3, \ldots\)
(C) Point \(P\) will be a point of minimum intensity if the path difference \(\Delta = (2n)\lambda/2\) for \(n = 0, 1, 2, 3, \ldots\)
(D) there is no such condition for the path difference as there is no interference
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Q.45 Two point charges (Q each) are placed at (0, y) and (0, –y). A point charge q of the same polarity can move along X axis. Then,
(A) the force on q is maximum at \( x = \pm \frac{y}{\sqrt{2}} \)
(B) the charge q is in equilibrium at the origin
(C) the charge q performs an oscillatory motion about the origin
(D) The equilibrium is not a stable equilibrium
Sol. [A, B, D]
At a point with coordinates (x, 0) the force is
\[ F = \frac{2Qq}{4\pi \varepsilon_0 \left( x^2 + y^2 \right)^{3/2}}. \]
For \( F \) to be maximum, equating \( \frac{dF}{dx} \) to zero gives \( x = \pm \frac{y}{\pi}. \) The
charge is obviously in equilibrium at the origin. However, the equilibrium is not stable since the force is repulsive and hence will not be able to restore the charge at the origin. The charge therefore cannot perform oscillatory motion.

Q.46 The network of identical resistors as shown between points A and B is connected to a dc source of emf V. Then,
(A) potential at point D is V/2
(B) current between points A and C is the same as that between F and B
(C) current between C and D is half that between points C and F
(D) current between points E and F is one third that between F and B
Sol. [A, B, C, D]
Note that points D and E are coincident and the circuit is symmetric about this point. Consider a current \( i \) entering the circuit at point A and leaving it at point B. If current \( i_1 \) flows between A and D, a current \( (i - i_1) \) flows between A and C. By symmetry current between D and B is \( i_1 \) and that between F and B is \( (i - i_1) \). The current \( (i - i_1) \) gets branched at C and \( (1/3) (i - i_1) \) flows between C and D and continues upto F. This is because there is no branching of current at D. Obviously \( (2/3) (i - i_1) \) flows between C and F. Note that total resistance between C and F is half of that along the path ADF.

Q.47 A concave lens is placed in the path of a uniform parallel beam of light falling on a screen as shown. Then,
(A) intensity of light will be the same everywhere on the screen
(B) intensity in region AB will be smaller than what is would be in the absence of the lens
(C) in the region AC and BD, the intensity will be smaller than what it would be in the absence of the lens
(D) in the region AC and BD, the intensity will be greater than what is would be in the absence of the lens
Sol. [B, D]
Note that the intensity of light in the region AB (when the lens is absent) now gets distributed over the region CD. In the regions AC and BD light intensity is due to both the direct beam and the diverged light from the lens.

Q.48 A hydrogen atom and a doubly ionized lithium atom are both in the second excited state. If \( L_H \) and \( L_{Li} \) respectively represent their electronic angular momenta and \( E_H \) and \( E_{Li} \) their energies, then -
(A) \( L_H > L_{Li} \) and \( |E_H| > |E_{Li}| \)
(B) \( L_H = L_{Li} \) and \( |E_H| < |E_{Li}| \)
(C) \( L_H = L_{Li} \) and \( |E_H| > |E_{Li}| \)
(D) \( L_H < L_{Li} \) and \( |E_H| < |E_{Li}| \)
Sol. [B]
For both the atoms the second excited state corresponds to \( n = 3 \). Therefore, the angular momentum for each of them is \( 3(h/2\pi) \). The energy, however, is proportional to \( Z^2 \) where \( Z \) is the atomic number and hence numerical value of energy for hydrogen is less than that for lithium.

Q.49 Refer to the circuit diagram and the corresponding graph. The current rises when key K is pressed. With \( R = R_1 \) and \( L = L_1 \) the rise of current is shown by curve (1), while curve (2) shows the rise of current when \( R = R_2 \) and \( L = L_2 \). Then,
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NSEP 2008-2009 EXAMINATION

PART B

Marks : 60

* All questions are compulsory.

* All questions carry equal marks

Q.50 Let the energy, magnitude of linear momentum and angular frequency of an electron in hydrogen atom be E, p and \( \omega \) respectively. If \( n \) be the corresponding quantum number, then
(A) \( \frac{E}{\omega} \) varies as \( n \)
(B) \( \frac{E}{p} \omega \) is independent of \( n \)
(C) \( p \omega \) varies as \( n^{1/2} \)
(D) \( E p \omega \) is independent of \( n \)

Sol. [A, B]

Note the proportionalities: energy \( E \propto \frac{1}{n^2} \), angular momentum \( L \propto n \), radius of orbit \( r \propto n^2 \), periodic time \( T \propto n^3 \), angular frequency \( \omega \propto \frac{1}{n^2} \) and speed of electron in an orbit \( v \propto \frac{1}{n} \) and get the result.

Q.51 A solid wooden cube of side \( l \) and mass \( M \) is resting on a horizontal table, as shown in the figure. The cube is constrained to rotate about an axis through \( D \) and perpendicular to the face ABCD. A bullet of mass \( m \) moving with speed \( v \) strikes the block at a height \( \frac{2l}{3} \) as shown. Let the line along which the bullet moves be in the plane passing through the centre of mass of the block and parallel to the face ABCD. Determine the minimum value of \( v \) that topples the block.

Sol. Let \( \omega \) be the angular velocity of the cube (just after the bullet strikes) about an axis passing through \( D \). Conservation of angular momentum about this axis gives \( mv(2l/3) = I \omega \) where \( I \) is the moment of inertia about the axis through \( D \). This is \( [(M l^2/6) + (M l^2/2)] = (2M l^2/3) \) From the above equation \( \omega = (mv/M l) \). The cube will topple if the centre of mass is just able to rise from \((l/2)\) to \((l/2)\). In such a case, the rotational energy must be equated to the change of potential energy. Thus, \( \frac{1}{2} I \omega^2 = M g \left( \frac{l}{2} - \frac{l}{2} \right) \).

Using the values of \( I \) and \( \omega \), we get the expression for \( v \) that will just topple the cube:

\[
\frac{v}{m} = \frac{M}{3g} \left[ \frac{(\sqrt{2} - 1)}{2} \right]^{1/2}
\]

Q.52 Consider flow of heat from an inner sphere of radius \( a \) at temperature \( T_2 \) to an outer concentric sphere of radius \( b \) at temperature \( T_1 \) through an insulating material filled in between the two spheres. Obtain expressions for the total heat current \( H \) and the temperature \( T \) at a distance \( r \) from the centre, when a steady state is reached.

Sol. Consider a concentric spherical shell of radius \( r \) and thickness \( dr \) where \( a < r < b \). Let \( k \) be the coefficient of thermal conductivity and \( \frac{dT}{dr} \) be the temperature gradient at this shell. Then, the heat current (that is the amount of heat flowing per unit time) is given by

\[
H = -4\pi r^2 k \frac{dT}{dr}
\]

where the negative sign indicated a fall of temperature with increase of radius. From this we get. Integrate this between limits \( a \) and \( b \) for \( r \) and between \( T_2 \) and \( T_1 \) for \( T \). This gives an expression for the heat current. Now, use to find the indefinite integral with a constant of integration \( c \). The constant of integration \( c \) comes out to be where temperature \( T_2 \) corresponds to radius \( a \). Using this we write an expression in terms of temperature \( T \) at a radius \( r \) in general. Solving this we get an expression for the temperature \( T \) at a distance \( r \) from the centre.
Q.53 A double convex lens of radii of curvature 10 cm (µ = 1.5) and a double concave lens of radii of curvature 12 cm (µ = 1.6) are separated by a liquid of refractive index 1.2, as shown. Find the effective focal length of the combination.

Sol. The convex lens with a plano-concave liquid lens on left can be taken to form one group of lenses. Similarly, a plano-convex liquid lens with the concave lens can be taken to form another group of lenses. The liquid layer of width 4.8 cm in between will then be equivalent to an air slab of thickness \((4.8 \times 1.2) = 5.76\) cm. Note that this distance \(d\) is the distance of separation between the two lens combinations described above. The focal length of the given convex lens can be determined using lens maker's formula and it comes out to be 10.0 cm. Similarly the focal length of the adjacent plano-concave liquid lens turns out to be \((- 50.0)\) cm. The focal length of this combination 12.5 cm. Use the relation, in this case the distance of separation \(d\) is zero. Similarly, the focal lengths of the plano-convex liquid lens and the concave lens on right come out to be 60.0 cm and \((- 10.0)\) cm respectively. The focal of their combination turns out to be \((- 12.0)\) cm. Now, the two lens combinations are of focal lengths 12.5 cm and \((- 12.0)\) cm separated by a distance of 5.76 cm. The relation written above can again be used to get the effective focal length as 28.49 cm and acts as a converging lens.

Q.54 A material with uniform resistivity \(\rho\) is formed in the shape of a wedge as shown. Determine the resistance between face A and face B of this wedge.

Sol. Refer to the figure in the question paper. Let the front lower edge of the wedge to be along X axis so that left extreme of the edge can be taken to be at \(x = 0\) and the other extreme to the right to be at \(x = L\). Choose an element of the wedge perpendicular to this edge at a distance \(x\) and of width \(dx\). This section will have a height, so that the area of cross section of this element will be \(WX\). Now, the resistance of this element can be written as \(dR\). Integrate this between \(x = 0\) and \(x = L\). The expression for the resistance of the wedge comes out to be \(R\).

Q.55 Show that the capacitance of parallel plate capacitor filled with a dielectric whose dielectric constant increases linearly from one plate to the other, is

\[C = \frac{\varepsilon_0 A (K_2 - K_1)}{d \ln (K_2/K_1)}\]

where \(A\) is the area of each of the plates, \(d\) is the separation between them, \(K_1\) and \(K_2\) are dielectric constants near plate 2 respectively.

Sol. Let the relation for variation of dielectric constant be \(K = mx + K_1\) where \(x\) is the distance from plate 1. At \(x = 0\), that is, at plate 1, \(K = K_1\) while at \(x = d\), \(K = K_2\). With this we get \(m = (K_2 - K_1)/d\). Now, we use \(E = \varepsilon_0 E_0\). The potential difference \(V\) between the plates of the capacitor can be determined by writing \(V = \int_{0}^{d} dV = E dx\). Note that \(E\) is a function of \(x\). Limits of integration for \(x\) are 0 and \(d\). Then use \(C = Q/V\) where \(Q = \sigma A\). The final relation for the capacitance is \(C\).