CAREER POINT

[Class-X]
MATHEMATICS
SESSION:2017-18

Time allowed: 3 hrs. Maximum Marks : 80

General Instructions :

(i) All questions are compulsory.

(ii) This question paper consists of 30 questions divided into four sections –A, B, C and D

(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions

(v) Use of calculator is **not** permitted.

SECTION-A

Question numbers 1 to 6 carry 1 mark each.

Q.1 If x = 3 is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

Q.2 What is the HCF of smallest prime number and the smallest composite number ?

Q.3 Find the distance of a point P(x, y) from the origin.

Q.4 In an AP, if the common difference (d) = – 4, and the seventh term (a_7) is 4, then find the first term.

Q.5 What is the value of $(\cos^2 67^\circ - \sin^2 23^\circ)$?

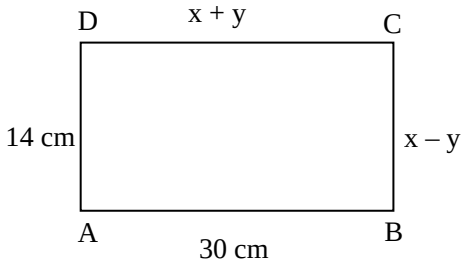
Q.6 Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$ then find $\frac{ar\,\Delta ABC}{ar\,\Delta PQR}$

SECTION-B

Question numbers 7 to 12 carry 2 mark each.

Q.7 Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Q.8 In Fig. 1.ABCD is a rectangle. Find the values of x and y.



Q.9 Find the sum of first 8 multiples of 3.

Q.10 Find the ratio in which P(4, m) divides the line segment joining the points A(2 ,3) and B(6, –3). Hence find m.

Q.11 Two different dice are tossed together. Find the probability:
(i) of getting a doublet
(ii) of getting a sum 10, of the numbers on the two dice.

Q.12 An integer is chosen at random between 1 and 100. Find the probability that it is :
(i) divisible by 8.
(ii) not divisible by 8.

SECTION-C

Question numbers 23 to 22 carry 3 mark each.

Q.13 Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

Q.14 Find all zeroes of the polynomial $(2x^4 – 9x^3 + 5x^2 + 3x – 1)$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 – \sqrt{3})$.

Q.15 If A(–2,1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

OR

If A(–5,7), B(–4, –5), C(–1, –6) and D(4,5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

Q.16 A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km./h from the usual speed. Find its usual speed.

Q.17 Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

OR

If the area of two similar triangles are equal, prove that they are congruent.

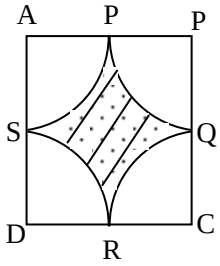
Q.18 Prove that the lengths of tangents drawn from an external point to a circle are equal.

Q.19 If $4 \tan \theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta + 1}\right)$

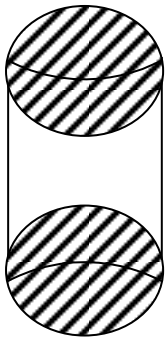
OR

If $\tan 2A = \cot (A - 18^{o})$, where 2A is an acute angle, Find the value of A.

Q.20 Find the area of the shaded region in Fig, 2, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD, and DA respectively of a square ABCD of side 12 cm. [Use $\pi = 3.14$]



Q.21 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 3. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article



OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. find the volume of the rice. How much canvas cloth is required to just cover the heap ?

Q.22 The table below shows the salaries of 280 persons :

Salary (In thousand Rs.)	No. of Persons
5–10	49
102–15	133
15–20	63
20–25	15
25–30	6
30–35	7
35–40	4
40–45	2
45–50	1

Calculate the median salary of the data.

SECTION-D

Question numbers 23 to 30 carry 4 mark each.

Q.23 A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed ?

Q.24 The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15 Find the numbers.

Q.25 In an equilateral Δ ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC, Prove that $9(AD)^2 = 7(AB)^2$

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Q.26 Draw a triangle ABC with BC = 6 cm, AB = 5 cm and $\angle ABC = 60^o$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the Δ ABC.

Q.27 Prove that : $\frac{\sin A - 2\sin A}{2\cos^3 A - \cos A} = \tan A$

Q.28 The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic ? [Use $\pi = 3.14$]

Q.29 As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30º and 45º. If one ship is exactly behind the other on the same side of the light house, Find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

Q.30 The mean of the following distribution is 18. Find the frequency f of the class 19 – 21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

OR

The following distribution gives the daily income of 50 workers of a factory :

Dally Income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

SOLUTIONS

1. $x = 3$ is one root of the equation
$\therefore\ 9 - 6k - 6 = 0$
$\Rightarrow k = \frac{1}{2}$

2. The required numbers are 2 and 4.
HCF of 2 and 4 is 2.

3. $OP = \sqrt{x^2 + y^2}$

4. $a + 6(-4) = 4$
$\Rightarrow a = 28$

5. $\because\ \cos 67° = \sin 23^o$
$\therefore \cos^2 67^o - \sin^2 23 = 0$

6. $\frac{ar\,\Delta ABC}{ar\,\Delta PQR} = \frac{AB^2}{PQ^2}$
$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

7. Let us assume $5 + 3\sqrt{2}$ is a rational number.
$\Rightarrow 5 + 3\sqrt{2} = \frac{p}{q}$ where $q \neq 0$ and p q are integers.
$\Rightarrow \sqrt{2} = \frac{p - 5q}{3q}$
$\Rightarrow \sqrt{2}$ is a rational number as RHS is rational
This contradicts the given fact that $\sqrt{2}$ is irrational.
Hence $5 + \sqrt{2}$ is an irrational number

8. AB = DC and BC = AD
$\Rightarrow \left.\begin{matrix} \Rightarrow x + y = 30 \\ \text{and } x - y = 14 \end{matrix}\right\}$
Solving to get x = 22 and y = 8

9. $S = 3 + 6 + 9 + 12 + \ldots + 24$
$= 3(1 + 2 + 3 + \ldots + 8)$
$= 3 \times \frac{8 \times 9}{2}$
$= 108$

10. Let AP : PB = K : 1

$\therefore \frac{6k+2}{k+1} = 4$

$\Rightarrow$ k = 1, ratio is 1 : 1

Hence m = $\frac{-3+3}{2} = 0$

11. Total number of possible outcomes = 36

(i) Doublets are (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)

Total number of doublets = 6

$\therefore$ Prob (getting a doublet) = $\frac{6}{36}$ or $\frac{1}{6}$

(ii) Favourable outcomes are (4, 6) (5, 5) (6, 4) i.e., 3

$\therefore$ Prob (getting a sum 10) = $\frac{3}{36}$ or $\frac{1}{12}$

12. Total number of outcomes = 98

(i) Favourable outcomes are 8, 16, 24, ..., 96 i.e., 12

$\therefore$ Prob (integer is divisible by 8) = $\frac{12}{98}$ or $\frac{6}{49}$

(ii) Prob (integer is not divisible by 8) = $1-\frac{6}{49}$

= $\frac{43}{49}$

13. $404 = 2 \times 2\ 101 = 2^2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$\therefore$ HCF of 404 and 96 = $2^2 = 4$

LCM of 404 and 96 = $101 \times 2^5 \times 3 = 9696$

HCF $\times$ LCM = $4 \times 9696 = 38784$

Also $404 \times 96 = 38784$

Hence HCF $\times$ LCM = Product of 404 and 96.

14. $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

2 + $\sqrt{3}$ and 2– $\sqrt{3}$ are zeroes of p(x)

$\therefore$ $p(x) = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) \times g(x)$

$= (x^2 - 4x + 1)\, g(x)$

$(2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1$

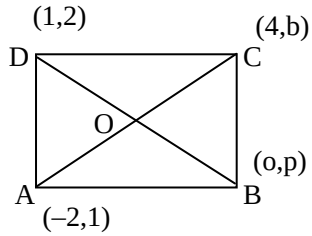
$\therefore g(x) = 2x^2 - x - 1$

$= (2x + 1)(x - 1)$

Therefore other zeroes are $x = -\frac{1}{2}$ and = 1

$\therefore$ Therefore all zeroes are $2 + \sqrt{3}$, $2 - \sqrt{3}$, $-\frac{1}{2}$ and 1

15. ABCD is a parallelogram
$\therefore$ diagonals AC and BD bisect each other
therefore



Mid point of BD is same as mid point of AC

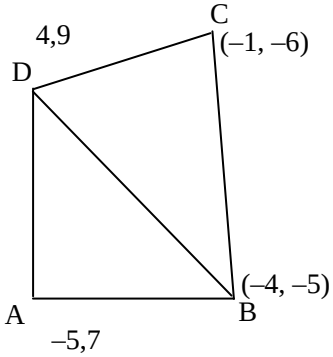
$\Rightarrow \left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{b+1}{2}\right)$

$\Rightarrow \frac{a+1}{2} = 1$ and $\frac{b+1}{2} = 1$

$\Rightarrow$ a = 1, b = 1 therefore length of sides are $\sqrt{10}$ units each.

OR

Area of quad ABCD = Ar ΔABD + Ar ΔBCD



Area of ΔABD = $\frac{1}{2}$ | (–5) (–5 –5) + (–4) (5–7) + (4) (7 + 5) |

= 53 sq units

Area of ΔBCD = $\frac{1}{2}$ | (–4) (–6 –5) + (–1) (5 + 5) + 4 (–5 + 6) |

= 19 sq units

Hence area of qad. ABCD = 53 + 19 = 72 sq unit

16. Let the usual speed of the plane be x km/hr

$\therefore \frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60}$

$\Rightarrow x^2 + 100x - 300000 = 0$

$\Rightarrow x^2 + 600x - 500x - 300000 = 0$

$\Rightarrow$ (x + 600) (x – 500) = 0

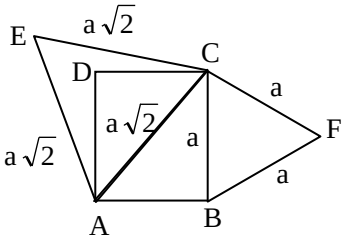
$x \neq -600$, $\therefore$ x = 500

Speed of plane = 500 km/hr

17. Let the side of the square be 'a' units

$\therefore AC^2 = a^2 + a^2 = 2a^2$

$\Rightarrow AC = \sqrt{2}$ a units



Area of equilateral $\Delta BCF = \frac{\sqrt{3}}{4}$ a^2 sq.u

Area of equilateral $\Delta ACE = \frac{\sqrt{3}}{4} (\sqrt{2}a)^2 = \frac{\sqrt{3}}{2}$ a^2 sq. u.

$\Rightarrow$ Area $\Delta BCF = \frac{1}{2}$ Ar ΔACE

OR

Let $\Delta ABC \sim \Delta PQR$.

$\therefore \frac{\text{ar } \Delta ABC}{\text{ar } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

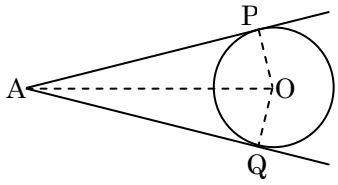
Given ar ΔABC = ar ΔPQR

$\Rightarrow \frac{AB^2}{PQ^2} = 1 = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

$\Rightarrow AB = PQ$, $BC = QR$, $AC = PR$

$\Rightarrow$ Therefore $\Delta ABC \cong \Delta PQR$ (sss congruence rule)

18. **Given :** Two tangents AP and AQ are drawn from a point A to a circle with centre O.



To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : AP is a tangent at P and OP is the radius through P.

$\therefore$ OP $\perp$ AP.

Similarly, OQ $\perp$ AQ.

In the right triangle OPA and OQA, we have

OP = OQ [radii of the same circle]

OA = OA [common]

$\therefore$ $\Delta OPA \cong \Delta OQA$ [by RHS–congruence]

Hence, AP = AQ

19. $4 \tan \theta = 3$

$\Rightarrow \tan \theta = \frac{3}{4}$

$\Rightarrow \sin \theta = \frac{3}{5}$ and $\cos \theta = \frac{4}{5}$

$$\therefore \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1}$$

$$= \frac{13}{11}$$

OR

$\tan 2A = \cot (A - 18^\circ)$

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow 3A = 108^\circ$

$\Rightarrow A\ 36^\circ$

20. Radius of each arc drawn = 6 cm

Area of one quadrant = $(3.14) \times \frac{36}{4}$

Area of four quadrants = $3.14 \times 36 = 113.04$ cm^2

Area of square ABCD = $12 \times 12 = 114$ cm^2

Hence Area of shaded region = $144 - 113.04$

$= 30.96$ cm^2

21. Total surface Area of article = CSA of cylinder + CSA of 2 hemispheres

CSA of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 3.5 \times 10$

$= 220$ cm^2

surface Area of two hemispherical scoops = $4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154$ cm^2

Total surface Area of article = $220 + 154 = 374$ cm^2

OR

Radius of conical heap = 12 m

Volume of rice = $\frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$ m^3 = 528 m^2

Area of canvas cloth required = $\pi r\ell$

$\ell = \sqrt{12^2 + (3.5)^2} = 12.5$ m

$= 471.4$m^2

22.

Salary (in thousand Rs.)	No. of persons (f)	cf
5 –10	49	49
10–15	133	182
15–20	63	245
20–25	15	260
25–30	6	266
30–35	7	273
35 –40	4	277
40 – 45	2	279
45 – 50	1	280

$\frac{N}{2} = \frac{280}{2} = 140$

Median class is 10 – 15

$\text{Median} = \ell + \frac{h}{f}\left(\frac{N}{2} - C\right)$

$= 10 + \frac{5}{133}(140 - 49)$

$= 10 + \frac{5 \times 91}{133}$

$= 13.42$

Median salary is Rs 13.42 thousand or Rs 13420 (approx)

23. Let the speed of stream be x km/hr.

$\therefore$ The speed of the boat upstream = (18 – x) km/hr
and Speed of the boat downstream = (18 + x) km/hr

As given in the question.

$\frac{24}{18-x} - \frac{24}{18+x} = 1$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow (x + 54)(x - 6) = 0$

$x \neq 54, \therefore x = 6$

$\therefore$ Speed of the stream 6 km/hr.

OR

Let the original average speed of train be x km/hr.

Therefore $\frac{63}{x} + \frac{72}{x+6} = 3$

$\Rightarrow x^2 - 39x - 126 = 0$

$\Rightarrow (x - 42)(x + 3) = 0$

$x \neq -3 \therefore x = 42$

Original speed of train is 42km/hr.

24. Let the four consecutive terms of the A.P. be
$a - 3d$, $a - d$, $a + d$, $a + 3d$.
By given conditions
$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$
$\Rightarrow 4a = 32$
$a = 8$
and $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$
$\Rightarrow 8a^2 = 128d^2$
$\Rightarrow d^2 = 4$
$\Rightarrow d = \pm 2$
$\therefore$ Numbers are 2, 6, 10, 14, or 14, 10, 6, 2.

25. Draw $AE \perp BC$
$\Delta AEB \cong \Delta AEC$ (RHS congruence rule)
$\therefore BE = EC = \frac{1}{2} BC = \frac{1}{2} AB$
Let $AB = BC = AC = x$
Now $BE = \frac{x}{2}$ and $DE = BE - BD$
$= \frac{x}{2} - \frac{x}{3}$
$= \frac{x}{6}$
Now $AB^2 = AE^2 + BE^2$ ....(1)
and $AD^2 = AE^2 + DE^2$ ....(2)

From (1) and (2) $AB^2 - AD^2 = BE^2 - DE^2$
$\Rightarrow x^2 - AD^2 = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{6}\right)^2$
$\Rightarrow AD^2 = x^2 - \frac{x^2}{4} + \frac{x^2}{36}$
$\Rightarrow AD^2 = \frac{28}{36} x^2$
$\Rightarrow 9AD^2 = 7AB^2$

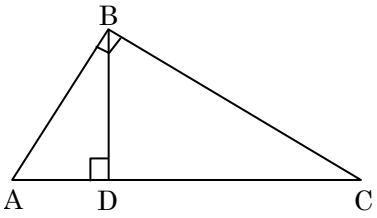
OR

Given : A right-angled triangle ABC in which $\angle B = 90^\circ$.

To Prove : (Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2.

i.e., $AC^2 = AB^2 + BC^2$

Construction : From B draw BD ⊥ AC.



Proof : In triangle ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90^o]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$

[$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow AB^2 = AD \times AC$ ....(i)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90^o]

and, $\angle C = \angle C$ [Common] So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow BC^2 = AC \times DC$ ....(ii)

Adding equation (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

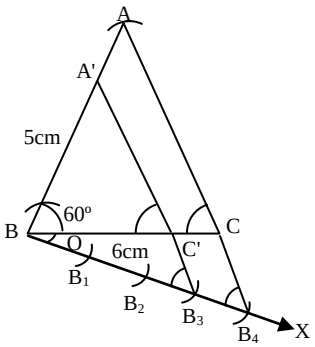
$\Rightarrow AB^2 + BC^2 = AC (AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = AC^2$

Hence, $AC^2 = AB^2 + BC^2$

26.



Steps of constructions

(1) Draw a ΔABC with side BC = 6cm, AB = 5 cm and $\angle ABC = 60^{\circ}$

(2) Draw a ray BX making an acute angle with BC on the opposite side of vertex A

(3) Mark 4 points B_1, B_2 B_3. B_4 (as 4 is greater in 3 and 4) on line BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

(4) Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'

(5) Draw a line through C' parallel to AC intersecting AB at A'. $\Delta A'BC'$ is the required triangle

27. $\text{LHS} = \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A\,(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

$= \dfrac{\sin A\,[1 - 2(1 - \cos^2 A)]}{\cos A - (2\cos^2 A - 1)}$

$= \tan A = \text{RHS}$

28. Here $r_1 = 15$ cm, $r_2 = 5$ cm and $h = 24$ cm

(i) Area of metal sheet = CSA of the bucket + area of lower end

$= \pi\ell\,(r_1 + r_2) + \pi r_2^2$

where $\ell = \sqrt{24^2 + (15-5)^2} = 26$ cm

$\therefore$ Surface area of metal sheet = $3.14\,(26 \times 20 + 25)$ cm^2

$= 1711.3$ cm^2

we should avoid use of plastic because it is non-degradable or similar value

29. Let AB be the tower and ships are at points C and D.

$\tan 45^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{AB}{BC} = 1$

$\Rightarrow AB = BC$

Also $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow AB + CD = \sqrt{3}AB$

$\Rightarrow CD = AB(\sqrt{3} - 1)$

$= 100 \times (1.732 - 1)$

$= 73.2$ m

30.

Class	**x**	**r**	**fx**	
11-13	12	3	36	
13-15	14	6	84	
15-17	16	9	144	
17-19	18	13	234	
19-21	20	f	20f	
21-23	22	5	110	For x
23-25	24	$\frac{4}{40+f}$	$\frac{96}{704+20f}$	$\frac{\sum f}{\sum fx}$

Mean $= 18 = \frac{704 + 20f}{40 + f}$

$\Rightarrow 720 + 18f = 704 + 20f$

$f = 8$

OR

Coumulative frequency distribution table of less than type is

Daily income	**Cumulative frequency**
Less than 100	0
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50